

# *Reading Comprehension in Problems Solving in Mathematics Test of Prova Brasil*

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## **Abstract:**

Difficulty in reading comprehension and problem solving presented in the results of large-scale assessment may be in basic cognitive processes of reading. Researches suggest the relation between reading comprehension skills and mathematical problem solving. The aim of this article is to present the articulation between reading and problem solving, considering the cognitive dimension, and to accentuate the importance of automaticity and reading comprehension for solving problems in the Mathematics test of Prova Brasil. We analysed the descriptors of three topics of the reference matrix of Portuguese Language that indicate the reading abilities in items of Prova Brasil of a standardized test, elaborated in order to emphasize the need for the student to have developed the processes of reading comprehension to be successful in solving mathematical problems.

## **Keywords:**

Reading comprehension. Problem solving. Prova Brasil.

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## INTRODUCTION

The results of external assessments, such as Prova Brasil (Avaliação Nacional do Rendimento Escolar – Anresc) [National Assessment of Student Performance – Anresc], a census evaluation that covers public school students from the initial and final years of elementary school, conducted by the National Institute of Educational Research Anísio Teixeira (INEP), as well as the results of the Programme for International Student Assessment (PISA), from the Organisation for Economic Cooperation and Development (OECD), an international assessment, show that the development of reading comprehension does not meet the expectations for students in the assessed grades.

Since reading involves decoding and comprehension processes, decoding processes with automaticity are necessary to understand the text. If decoding processes are not automated, comprehension is compromised, which can interfere with the results of large-scale evaluations. Assuming that reading comprehension is a predictor of problem solving, a skill assessed in the Mathematics test of Prova Brasil, the objectives of this article are to present the articulation between reading and problem solving, considering the cognitive dimension, and to highlight the importance of automaticity and reading comprehension for problem solving in the Mathematics test of Prova Brasil.

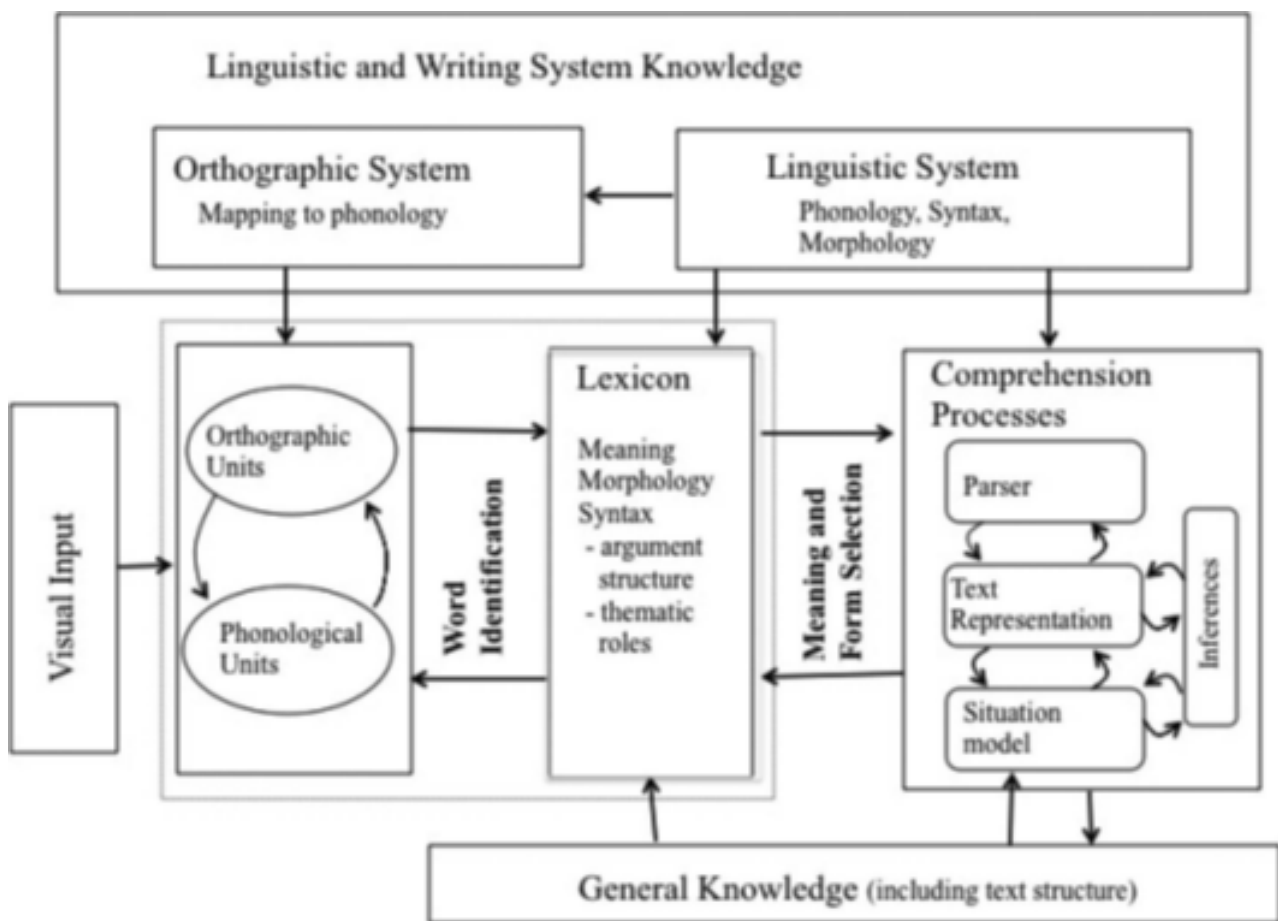
We analysed items of this external evaluation, considering the theoretical construct of reading comprehension (PERFETTI; LANDI; OAKHILL, 2013; PERFETTI; STAFURA, 2014) and problem solving (PÓLYA, 2006), and related the topics of the reference matrix of Portuguese language, which address the processes of reading comprehension with the processes of problem solving.

## READING AND READING COMPREHENSION

From the cognitive point of view, reading is a complex skill that involves several processes to achieve its goal: reading comprehension. Considering that reading involves the processes of decoding and comprehension, the processes of recognition of letters, syllables and printed words, which are on the surface of the text, are understood as decoding of the written word and as the specific mode of information acquisition (MORAIS, 1996; DEHAENE, 2012). This basic reading process transforms each visual stimulus (the written

word) through a series of stages, involving visual, phonological and lexical systems, and episodic memory (acquired knowledge, information) until it is finally understood in the semantic representation system. The processing that occurs at each stage is supposed to be learned and the degree of that learning is evaluated in relation to the use of attention in the development of reading (LABERGE; SAMUELS, 1974).

The comprehension processes (figure 1) involve the use of this information to build a representation consistent with the written text (SMITH, 2004; PERFETTI; LANDI; OAKHILL, 2013; PERFETTI; STAFURA, 2014). In this meaning-construction process, the reader who monitors their reading has a greater cognitive control of the process of reading comprehension (OAKHILL; HARTT; SAMOLS, 2005).



Source: Perfetti; Stafura (2014, p. 24).

**Figure 1** – Reading comprehension model

In this model of reading comprehension, the task of the proficient reader begins with the decoding of the visual input with automaticity. This model of reading comprehension is based on three assumptions:

- 1) Three classes of knowledge sources are used in reading: linguistic knowledge,

orthographic knowledge, and general knowledge (knowledge about the world, including knowledge of text forms, e.g., text genres).

2) **The processes of reading – decoding, word identification, meaning retrieval, constituent building (sentence parsing), inferencing, and comprehension monitoring – use these knowledge sources in both constrained ways** (e.g., decoding uses orthographic and phonological knowledge but not general knowledge) and in interactive ways (e.g., inferences use general knowledge and propositional meaning extracted from sentences).

3) These processes take place within a cognitive system that has pathways between perceptual and long-term memory systems and limited processing resources (PERFETTI; STAFURA, 2014, p. 24-25).

These assumptions are related to the automation in the decoding model by LaBerge and Samuels (1974) and in the alphabetical and linguistic knowledge and the spelling system, demonstrated by Morais (1996) and Dehaene (2012) as processes required for automaticity. In addition to these automaticity processes, reading comprehension involves other complex processes to construct meanings for words and adapt them to the context (PERFETTI; LANDI; OAKHILL, 2013), which require prior knowledge, syntactic-semantic knowledge and vocabulary knowledge. These processes of reading comprehension are integrated after the recognition of the written word (2013). Therefore, when the reader decodes a word with automaticity, they connect it to a mental representation coherent with the text that is constructed as the reading process is developed.

For the successful understanding of the text, the integration process addresses the construction of propositions that are coherent with the text (construction of inferences), the syntactic-semantic processing (parser) of the text and the monitoring of reading comprehension. These processes are successful when done accurately to the written word.

Beginning readers do not yet have the automated decoding processes, as they are undergoing the initial reading learning process (MORAIS; LEITE; KOLINSKY, 2013). However, students in the final cycles of elementary education who did not automate these processes are unsuccessful readers (MACHADO, 2018), because they present a pattern of reading behaviour in which cognitive resources are directed to the decoding of the written word and not the understanding of the text. Thus, the processes of constructing inferences for the comprehension of the text are not successful.

The skilled reader is a good “understander” of text because the pattern of reading behaviour points out that the reader has automated the processes of initial reading learning and directs attention to understanding and monitoring reading comprehension (MACHADO, 2018).

Considering that reading comprehension is basic for problem solving, the skilled reader recognizes the written word with automaticity, constructs meaning consistent with the text, and can succeed in solving mathematical problems.

## PROBLEM SOLVING

Problem solving is the application of mathematical concepts in the resolution of exercises (D'AMBROSIO, 2008). However, problem solving is a much more complex domain, “it means looking for and planning situations open enough to induce in students the search and appropriation of adequate strategies, not only to give answers to school questions but also to everyday reality ones” (ECHEVERRÍA; POZO, 1998, p.14, own translation)<sup>1</sup>.

Problems are solved every day, in several contexts and in varying circumstances. So, what do we mean by problem? We mean a conscious search for adequate strategies to reach a fundamental solution for human, social and educational life (ONUCHIC, 1999; ONUCHIC; ALLEVATO, 2005; PÓLYA, 2006; ALLEVATO; ONUCHIC, 2009).

Echeverría and Pozo (1998) suggest that problem solving should be based on school curriculum so that teachers from all areas of knowledge can work from a perspective of teaching problem situations not only to the contents of their disciplines, but to apply this knowledge to everyday life. In this perspective, problem “is any task or activity for which students do not have methods or techniques or prescribed or memorized rules, nor the perception that there is a specific method to get to the correct solution” (VAN DE WALLE, 2009, p. 57, own translation)<sup>2</sup>, but the ability to test strategies to solve the problem. Thus, problem solving objectives are not limited to the simple consolidation of the content learned in a discipline, such as Mathematics or Portuguese, but to lead the student to make a habit of challenging themselves, proposing problems and solving them as a form of learning.

However, the failure to solve the problem can be due to not understanding the content, the process or the resolution strategies or because the processes of reading comprehension have not been developed. Echeverría and Pozo (1998) and Van de Walle (2009) suggest that, to solve a problem, one must have developed the processes of reading comprehension that are related to linguistic knowledge (orthographic, morphological and syntactic-semantic), mathematical symbols and the content covered, in addition to developing strategies for solving the problem.

Mathematical problems can be divided into verbal ones, those in which predominate terms and expressions of mathematical language with relevant information in the form of a text (BIVAR; SANTOS; AIRES, 2010; PONTE; QUARESMA, 2012), and nonverbal

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<sup>1</sup> “significa procurar e planejar situações suficientemente abertas para produzir nos alunos uma busca e apropriação de estratégias adequadas não somente para darem respostas a perguntas escolares como também às da realidade do cotidiano” (ECHEVERRÍA; POZO, 1998, p. 14).

<sup>2</sup> problema “é qualquer tarefa ou atividade para a qual os estudantes não têm métodos ou técnicas ou regras prescritas ou memorizadas, nem a percepção de que haja um método específico para chegar à solução correta” (VAN DE WALLE, 2009, p. 57).

problems, in which prevail notations and mathematical formulas, followed by the commands “solve”, “calculate”, among others (TOOM, 2010).

In verbal problems, we can verify the relationship between reading and problem solving. In this type of problem, the reader needs to construct a coherent meaning with the written text to solve the problem. These problems may be: verbal problems of calculation (simple and compound standard problems, process problems); semantic and conceptual structures (comparison, combination and change or alteration); and multiple-choice items (alternatives with one correct answer) (CORREIA, 2013).

Knowing these types of problems and their objectives can help in the development of skills and competences to achieve success in problem solving, according to the teaching-learning approach:

- 1) teaching **about** problem solving: works with mathematical content using Pólya’s theory (understanding the problem, drawing up a plan, executing the plan and assessing the solution);
- 2) teaching **for** problem solving: uses mathematical knowledge to solve problems in any situation, be it school-based or not;
- 3) teaching **via or through** problem solving: Mathematics teaching focuses on problem solving, that is, the teacher stems from a problem to teach the content (SCHROEDER; LESTER, 1989, p. 32, own translation)<sup>3</sup>.

Thus, for the resolution of problems, it is necessary to construct meaning of mathematics. Learning mathematics means thinking mathematically, i.e. 1) developing a mathematical point of view – knowing the mathematical processes and knowing how to apply them; and 2) developing competence, through mathematical tools, to understand the structure – the construction of mathematical meaning (SCHOENFELD, 1992).

This construction of meaning is based on acquired mathematical knowledge, and building this knowledge is an active and reflective process of thinking. However, if the decoding processes are not automated for the cognitive resources to be directed to the comprehension processes, nothing happens (PERFETTI; STAFURA, 2014). Van de Walle (2009) suggests that all mathematical knowledge consists of representations of internal or mental ideas that we construct in our mind, so problem solving requires a combination of reading comprehension and math skills.

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<sup>3</sup> “1) ensinar **sobre** resolução de problemas: trabalha os conteúdos matemáticos utilizando a teoria de Pólya (compreender o problema, elaborar um plano, executar o plano e avaliar a solução);  
2) ensinar **para** resolver problemas: utiliza o conhecimento matemático para resolver problemas em qualquer situação, seja ela escolar ou extraescolar;  
3) ensinar **via ou através** da resolução de problemas: o ensino de Matemática tem como foco a resolução de problemas, ou seja, o professor parte de um problema para ensinar o conteúdo” (SCHROEDER; LESTER, 1989, p. 32).

## READING FOR PROBLEM SOLVING

From the cognitive point of view, problem solving is a complex activity, which requires from students the access to various knowledge, such as:

the linguistic and factual knowledge is required for the translation of the problem; the knowledge about schemas is required for integration of the problem; the knowledge of strategies is necessary for planning solution and the algorithmic knowledge is necessary for the implementation of the solution (MAYER, 1992, p. 149).

In addition to mobilizing this knowledge, in the model of Pólya (2006), problem solving involves four steps to be developed: comprehension, designing a plan, executing the plan and assessing the achieved solution. Thus, to solve mathematical problems, the first step to be overcome is comprehension.

Aside from requiring the development of processes of reading comprehension, automaticity in decoding and precision in the construction of meaning of written text, students also need to master mathematical language, which is “a symbolic system with its own symbols, which are related according to certain rules” (LORENSATTI, 2009, p. 90, own translation)<sup>4</sup>.

Considering that mathematical writing has its own characteristics, which makes use of symbols, letters and words that are combined to compose the mathematical problem (SMOLE; DINIZ, 2001), it requires of the student a specific reading processing of the area. Thus, Mathematics classes, from the initial grades, should contemplate problem solving, allowing students to become familiar with language and symbols, perceiving the meaning of what they read and understanding their meanings in mathematical texts.

For problem solving, students need to identify variables and unknowns. If these are not explicit, procedures or algorithms will be needed to determine the solution to the problem. In some situations, it is only necessary to rearrange the presentation of the information. However, in other more complex situations, solving problems implies identifying information and establishing relationships between linguistic and mathematical knowledge.

In these more complex situations, studies point to the relationship between proficiency in reading and performance in mathematics, confirming our premise that reading comprehension is basic to mathematical problem solving. Santos (2009) verified that linguistic knowledge influences the information processing of the wording of mathematical problems, so the lack of such knowledge makes it difficult to solve the problem. Comério (2012) analysed the relationship between reading and problem solving of students in the 5th year

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<sup>4</sup> “um sistema simbólico, com símbolos próprios, que se relacionam segundo determinadas regras” (LORENSATTI, 2009, p. 90).

of elementary school in a public school in Campinas, São Paulo, and found that students who performed well in reading also performed well in problem solving. By investigating the effects of syntactic processing associated with other sources of information with Portuguese students in the 4th, 6th and 9th years of elementary education, Correia (2013) found that student performance does not only lie in mathematical strategies and procedures, although it plays an important role, but it is associated with the understanding of the wording of mathematical problems. Thus, she identified that linguistic structures of the statement were presented as a determining factor of difficulty for the students to access the mathematical knowledge. Fuchs et al. (2015) analysed the effects of reading comprehension on word problems, a type of text that describes a problem situation and which is converted into mathematical sentences that represent a problem in finding a solution. After performing the process of converting the mother language into mathematical language, students can perform the algorithms to present the answer to the problem. They concluded that the language comprehension of the verbal problem influenced the elaboration of strategies to solve a mathematical problem. Barcellos (2017) carried out three experiments to investigate the difficulties related to linguistic complexity in the wording of mathematical problems on numerical division, whose results point out the need for an interdisciplinary work between Portuguese and Mathematics. One of the recommendations of the researcher is to consider the wording of mathematical problems as a textual genre to be discussed in class, paying attention to the language structures. It suggests that this work should be carried out in partnership between these two subjects.

These studies corroborate our assertion that reading comprehension is basic to solving mathematical problems. With the proposal to prove this assertion, we identified the topics of the Portuguese reference matrix, which deal with the processes of reading comprehension, and show the need to master these processes in the resolution of items in the Mathematics test of Prova Brasil.

## **READING IN PROBLEM SOLVING IN THE MATHEMATICS TEST OF PROVA BRASIL**

Problem solving is “the driving force behind the development of mathematical science” (PONTE, 1992, p. 95, own translation)<sup>5</sup>. We also consider the importance attributed to problem solving in Mathematics teaching and learning and we believe that problem solving enables students to mobilize mathematical knowledge and to use strategies to achieve the result. In addition to developing this ability to manage information and mobilize stored knowledge, problem solving needs other skills, being a complex process that involves

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<sup>5</sup> Resolução de problemas é “a força motora do desenvolvimento da ciência Matemática” (PONTE, 1992, p. 95).



cognitive resources that are driven to find alternative solutions. According to model of Pólya (2006), whose problem solving is composed of four stages: 1) comprehension, 2) designing a plan, 3) implementing the plan, and 4) assessing; if the student is not successful at this first step (comprehension), they will not be able to move on to the others.

The first step, that of comprehension, anticipates that the student processes the problem. This implies recognizing automaticity in written information, mobilizing linguistic and syntactic-semantic knowledge, and constructing inferences consistent with the written text, in short, understanding the text. After understanding the written text, the next steps are related to the mobilization of mathematical knowledge. The second step is designing a plan. At this point, the student needs to check what procedures or strategies are needed (rules, algorithms or operations) to decompose the problem, look for similar problems, verify what is known and organize the problem data alongside the knowledge stored in memory. The third step, the execution of the plan, is when the student develops the previously prepared plan, making use of rules or procedures selected to reach the solution. The fourth and last stage, the assessment, is when the student verifies the whole process of solving the problem, that is, evaluates if they have reached the goal or if there are other strategies to be used. It is a process of metacognition in which there is awareness of the process of problem solving.

In order to solve a problem, students do not only depend on the skills to perform operations and apply mathematical concepts, but also on the reading skills that allow them to understand the wording of the problem and to evaluate (monitor) if the strategies chosen solve the problem. In this regard, we reaffirm that the first phase of Pólya's model (2006), comprehension, requires automated reading skills to direct cognitive resources for comprehension, only then mobilizing the other stages of the problem-solving model.

In order for the students to know what strategies to use to solve a problem, they must overcome the first stage of this model, reading comprehension, which requires lexical, linguistic and syntactic-semantic knowledge. Thus, problem solving requires that students have developed reading comprehension and mathematical knowledge (ECHEVERRÍA; POZO, 1998). These authors ratifies Pólya's problem solving model (2006), reinforcing the need for reading comprehension to solve mathematical problems.

Thus, understanding the wording of a mathematical problem and converting what is read into mathematical sentences (algorithms or operational strategies) are only possible with reading processing. The initial linguistic and semantic knowledge is processed and stored to be later transformed into mathematical information. For the student to construct this mathematical representation, they need to understand what they have read in the wording of the problem and choose, among the mathematical concepts they know, what will be the operation or content used to solve this problem.

Another type of knowledge needed to solve problems is the schematic, which refers to the prior knowledge that the student brings to the classroom (ECHEVERRÍA; POZO,

1998). This knowledge, too, is necessary in reading. In reading comprehension, the construction of representations of the text occurs from the convergence between the construction of meaning of the written text and the previous knowledge of the reader. Therefore, the text processing is performed during the reading and the reader constructs propositions of the main ideas, taken from the processed information of the written text, for the construction of the overall idea of the text. This process is carried out by means of inferences, which are the integration of the representation of the situation described by the written text with the previous knowledge relevant to the construction of meaning (KINTSCH; RAWSON, 2013). The construction of meaning of the text is only possible if there is this processing of textual information related to the prior knowledge of the reader.

In Pólya's resolution model (2006), the reader is expected to be proficient. Proficient (or skilled) readers already have the automated decoding process, being able to use cognitive resources in the processing of comprehension (MORAIS, 1996). Only with the automaticity in decoding and with cognitive resources directed to the comprehension of the text, the mathematical processes are triggered for the resolution of problems. Thus, understanding the mathematical problem can become difficult for unsuccessful readers (MACHADO, 2018), since they need to transform linguistic knowledge into mathematical propositions appropriate to the knowledge of mathematical vocabulary, which includes words that are specific to the components and processes of mathematics. The lack of automaticity in the decoding processes becomes a limiting factor in reading comprehension and consequently in problem solving.

Therefore, in the theoretical problem-solving framework of Pólya (2006) and Echeverría and Pozo (1998), the reading comprehension stage is a prerequisite for the other stages of problem solving, which involve mathematical knowledge. This model makes it possible to relate reading comprehension to the resolution of mathematical problems. For this, we use the reference matrix of the Portuguese test of Prova Brasil, whose reading conception is reading comprehension.

In order to confirm the relationship between reading comprehension and problem solving, we developed a standardized test according to the Mathematics test of Prova Brasil (MATOS, 2018), based on items available on the website of the Inep platform Devolutivas Pedagógicas (Pedagogical Feedback). We present question 4 of the standardized test of the 6th year of Mathematics (figure 2).

Marta bought hair ties and distributed among her 5 friends. Each one got 4 and still 3 hair ties were left. How many hair ties did Marta buy?  
(A) 17  
(B) 19  
(C) 20  
(D) 23

Propositions:

P1 - Distributed among her 5 friends (shared)

P2 - Each one got 4 hair ties (received from the division)

P3 - Still 3 hair ties were left (the rest to be added)

P4 - How many ties did Marta buy? (the question of the problem)

Expression:

$$X=5x4 + 3$$

$$X= 23$$

Source: Devolutivas Pedagógicas platform (own translation).

**Figure 2** – Question 4 of the standardized test of the 6th year of Mathematics and representation of the mathematical propositions of the problem

The problem in figure 2 illustrates the process of reading comprehension for problem solving: initially, the student needs to transform linguistic knowledge into a mental representation of the text and present the mathematical propositions in order to be able to answer the question of the problem. For this, the reader must associate each proposition with a mathematical procedure, such as “distribute”, which means “divide” in mathematical language, in order to define the subsequent mathematical procedures.

Another factor that contributes to the comprehension of the mathematical text is the ability to make inferences. The skilled reader makes inferences by reading the written text and these inferences are constructed in two ways: by relating the elements of the text (relating a pronoun to its antecedent) or by assigning a meaning that is not explicit (inferring that an action described in the text was performed in a certain way, for example, “going to school” implies understanding that he woke up early, got on the bus and went to school). Thus, the inability to make inferences limits the processing of text comprehension and hence of transforming inferences into mathematical propositions to solve the problem.

The unsuccessful reader may have different goals when reading the text because they direct their attention to the processes of decoding the written word and the cognitive resources do not turn to the comprehension of the text, thereby the processes of inference are not coherent with the written text.

Another example that demonstrates the relationship between reading comprehension and problem solving is expressed in question 4 of the standardized 9th year Math test (figure 3).

### HAPPY CIRCUS

**ENTRY PRICE: R\$ 10,00**

**BIG SALE FROM TUESDAY TO FRIDAY: 40% DISCOUNT.**

**STUDENTS: 50% DISCOUNT ON THE PRICE AT THE DATE.**

Maria, who is a student, went to the circus on a Saturday.

Therefore, she paid for the ticket:

(A) R\$ 5,00

(B) R\$ 6,00

(C) R\$ 8,00

(D) R\$ 9,50

Inferences:

I1 - Maria is a student (explicit inference)

I2 - Regardless of the day, the student is entitled to a 50% discount (explicit)

Propositions:

P1 - Entry price R\$ 10,00

P2 - 40% discount from Tuesday to Friday (division)

P3 - 50% discount for students (division)

P4 - Maria is a student

P5 - Maria went to the circus on a Saturday

P6 - How much did she pay for the ticket? (the question of the problem)

Expression:

$10 \times 50\%$

$10 \times 50 / 100$

$10 \times 1 / 2 = R\$ 5,00$

Source: Devolutivas Pedagógicas platform (own translation).

**Figure 3** – Question 4 of the standardized test of 9th year Mathematics, representation of the mathematical propositions of the problem.

The problem in figure 3 illustrates the process of reading comprehension for problem solving: the student needs to make inferences and relate them to mathematical propositions based on the mental representation of the text in order to be able to answer the question of the problem. For this, the reader needs to construct meaning consistent with the explicit and implicit inferences of the text. In this example, the reader needs to make inferences that are explicit in the text, relating that Maria is a student and that student has a 50% discount regardless of the day.

These processes of representation of linguistic knowledge and making inferences lead to the construction of meanings for words that are appropriate to the context, analyse sequences of words in their constituents (sentences) and provide the inferential integration of sentence information into more complex representations of texts (meaning construction). These representations do not result from exclusively linguistic processes but are intensified

by other sources of knowledge. Thus, the situational model (prior knowledge) brings the stored mathematical knowledge to the comprehension of the problem.

At this level of complexity of reading processing, processes that convert the sentence into semantic content or into its propositional meaning require syntactic knowledge. This way, the parser (syntactic sentence processing) identifies the information the reader uses when reading and determines how to relate it to the available information. The knowledge of a variety of syntactic structures contributes to not loading the memory in the processing of more complex structures.

Syntactic processing is necessary for the understanding of any written text, “processes that convert the sentence into semantic content or into its propositional meaning [...] require knowledge about syntactic forms and knowledge of words” (PERFETTI; LANDI; OAKHILL, 2013, p. 255, own translation)<sup>6</sup>. In problem solving, if the reader is aware of the syntactic structure in the construction of the mathematical problem, they are able to identify the complements of the sentence and relate them to the mathematical propositions more quickly, automating the process of comprehension. If the student understands the syntactic structure of the sentence “Marta bought hair ties and distributed among her 5 friends”, that is, someone practiced the action of “buying” alongside “distributing” and the action of “distributing” is the word that changes the previous situation of the subject to the other that establishes the mathematical relations proposed in the problem, so the student is able to select the corresponding mathematical proposition. To understand the problem, it is necessary to know that the syntactic structure of the verb and its quantifier (her 5 friends) activates the mathematical propositions related to the resolution of the problem.

By relating the processing of reading comprehension with problem solving, we have identified that, for the student to perform mathematical procedures, it is first necessary to understand the reading of written text. In the reference matrix of Portuguese language of Prova Brasil, three topics deal with the processes of reading comprehension and can give clues to this relationship of interdependence between reading comprehension and performance in problem solving.

The Reference Matrix of Portuguese Language focuses on reading practices, which are organized in two fields of competence: domain of reading strategies of different textual genres, which refers to topics 1, 2 and 3; and domain of linguistic-discursive resources in the construction of genres, discussed in topics 4, 5 and 6.

The topics in which we can check the components of reading comprehension processing that are needed for problem solving are topics 1, 4 and 5 of the 5th and 9th

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<sup>6</sup> “os processos que convertem a sentença em conteúdo semântico ou em seu significado proposicional [...] exigem conhecimentos sobre formas sintáticas e o conhecimento das palavras” (PERFETTI; LANDI; OAKHILL, 2013, p. 255).

grades of elementary school matrices and their respective descriptors that address the comprehension processes, as detailed in chart 1:

**Chart 1** – Topics and descriptors of the Reference Matrix of Portuguese Language of the 5th and 9th years of elementary school

Skills Object of knowledge	Competences developed by students
Topics	Descriptors
<b>I- Reading Procedures</b>	D1 – Find explicit information in a text. D3 – Infer the meaning of a word or expression. D4 – Infer implicit information in a text.
<b>IV- Coherence and Cohesion in Text Processing</b>	D2 – Establish relations between parts of a text, identifying repetitions or substitutions that contribute to the continuity of a text. D8 – Establish a cause/consequence relationship between parts and elements of the text. D12 – Establish logical-discursive relations present in the text, marked by conjunctions, adverbs, etc.
<b>V- Relations between Expressive Resources and Effects of Meaning</b>	D14 – Identifying the effect of meaning that results from the use of punctuation and other notations. D18 – Recognize the effect of meaning due to the choice of a particular word or expression. D19 – Recognize the effect of meaning that results from the exploitation of orthographic and/or morphosyntactic resources.

Source: Brasil (2013, p. 8-10, own translation).

In topic I, Reading Procedures, the descriptors present the linguistic abilities considered fundamental for the reading of texts. The skilled reader should be able to perform tasks such as locating information that is expressed in the text and identifying the overall meaning of that text. Moreover, they must know the meaning of a word or expression through the construction of inferences. In this topic, it is verified if the student is able to locate information that is on the surface of the text, but also if they are capable of constructing more elaborate meaning.

In topic IV, Coherence and Cohesion in Text Processing, the descriptors associated with this topic indicate the competence of the reader in recognizing the function of linguistic elements that signal references between the terms of the text that constitute textuality. This relates to those elements that build the articulation between the different parts of a text, constituting cohesion and coherence. Considering that coherence is the logical organization of the ideas of the text, for consistency it is necessary that the idea presented is related to the overall meaning of the text, respecting a progression of ideas. Koch and Travaglia (1989) explain that coherence is not on the linguistic surface but on the formal and content connections between the sequential elements of the text that are related to one another and facilitate the understanding of the text. And cohesion appears on the surface of the text by means of linguistic elements, expressing itself in its successive organization, in a linear way

(KOCH; TRAVAGLIA, 1989). Cohesion is the link between the ideas of the text, made through pronouns, conjunctions and prepositions, to promote the overall meaning of the text. Thus, the skills verified by the descriptors that comprise this topic require that the reader has linguistic knowledge to understand the text as a harmonious set of relations between the parts.

In topic V, Relations between Expressive Resources and Effects of Meaning, the descriptors deal with the meanings expressed in a text that result from the use of certain grammatical or lexical resources. Punctuation marks and other notations may express different meanings, depending on the author's intent and the context in which they are used. Expressive resources, too, are widely used and require the reader to pay close attention to the meaning effects underlying the text.

Hereinafter we analyse the descriptors of these three Portuguese language topics that indicate the reading abilities in questions of the Mathematics test of a standardized test, designed to highlight the need for the student to have developed the processes of reading comprehension for mathematical problem solving.

In topic I, the descriptor 1, "Finding explicit information in a text", shows that the reader needs to know how to locate explicit information. Finding explicit information on the surface of the text is considered the most elementary skill for developing other reading comprehension skills. This ability, too, is necessary for the processing of reading comprehension in mathematical problem solving. In the mathematical question 19 of the 6th year standardized test, we can identify this descriptor to check the reading skills needed to solve the problem (figure 4).

Carlos kept his soda cans collection in boxes. Each box fit 28 cans. He used 7 boxes and 6 cans were left. How many cans has Carlos' collection?

- (A) 194
- (B) 196
- (C) 202
- (D) 238

Inferences:

- I1- He kept the collection in boxes.
- I2- Each box fit 28 cans.
- I3- He used 7 boxes.
- I4- There were 6 cans left.

Propositions:

- P1 - Carlos stored his soda collection in boxes. (Division)
- P2 - Each box fit 28 cans.
- P3 - He used 7 boxes. (Multiply)
- P4 - There were 6 cans left. (Sum)
- P5 - How many cans does Carlos' collection have? (The question of the problem)

Source: Devolutivas Pedagógicas platform (own translation).

**Figure 4** – Question 19 of the standardized test of 6th year Mathematics

In the process of locating the words of the statement (text of the question), the student should be able to resume the text, locating, among other information, the one that was requested. Thus, the student has the task of locating the clues provided by the text (saved the collection in boxes, 28 cans fit in each box, 7 boxes were used, 6 cans left), assemble the algorithm and resume the text to locate the question that was asked (how many cans?) to get the correct answer.

Still in topic I, the descriptor 4, “Inferring implicit information in a text”, refers to the student’s reading ability to construct the meaning from inferences that are not in the text. Inferring means reasoning based on information already known in order to process new information that is not explicitly marked in the text. It is not, however, a question of verifying whether the student knows a word in the dictionary, but rather whether the sense activated corresponds to the one employed in the context and whether the student can identify the corresponding mathematical propositions. Therefore, the student transforms the linguistic knowledge into mathematical propositions to solve the question.

We can identify this reading process in question 18 of the standardized test of Mathematics of the 6th year, expressed in figure 5.

<p>A physical education teacher has 240 students. He notices that 50% of them know how to play volleyball. How many students of this group knows how to play the sport?</p> <p>(A) 100 (B) 120 (C) 160 (D) 190</p> <p>Inferences: I1- The teacher has 240 students. I2- Half the students do not know how to play volleyball. I3- The other half of the students knows how to play volleyball.</p> <p>Propositions: P1 - A physical education teacher has 240 students. P2 - 50% of them know how to play volleyball. (Half) P3 - How many students of this group knows how to play this sport? (The question of the problem)</p>
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Source: Devolutivas Pedagógicas platform (own translation).

**Figure 5** – Question 18 of the standardized test of 6th year Mathematics

In the construction of meaning of the text, the implicit information is those that are not clearly present in the text but can be retrieved by the student by making inferences from the linguistic expressions of the text. Thus, the student, to solve the mathematical problem of question 18, must infer that half the class does not know how to play volleyball, because he understands that 50% means half.



In topic I, descriptor 3, “Inferring the meaning of a word or expression”, the ability to construct inferences is verified in reading, that is, if the student knows how to construct, based on context, the meaning of a word or expression consistent with the written text. The student must recognize, among some possibilities, the one that corresponds to the meaning of the text read. In question 3 of the standardized Mathematics test of the 6th year, we can identify this reading process, expressed in figure 6.

<p>In order to buy candy, Leila traded a 2 reais bill for 50 cent coins. How many coins did she receive?</p> <p>(A) 4 coins (B) 5 coins (C) 10 coins (D) 20 coins</p> <p>Inferences: I1- Leila exchanged a 2 reais bill. I2- For coins of 50 cents.</p> <p>Propositions: P1 - Leila exchanged 2 reais bill. (Converted) P2 - For 50 cent coins. (Difference of cents and reais) P3 - How many coins did she receive? (The question of the problem)</p>
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Source: Devolutivas Pedagógicas platform (own translation).

**Figure 6** – Question 3 of the standardized test of 6th year Mathematics

In question 3, the ability to infer the sense of words, selecting information present on the surface of the text (notes and currencies, reais and cents) and establishing relations with previous knowledge (the Brazilian monetary system), makes it possible to identify the mathematical proposition necessary to resolve the issue, the amount received in the exchange of banknotes for coins.

In topic IV, descriptors 2 and 12, “Establish relations between parts of a text, identifying repetitions or substitutions that contribute to the continuity of a text” and “Establish logical-discursive relations present in the text, marked by conjunctions, adverbs, etc.” are exemplified in question 17 of the standardized 6th year Mathematics test (figure 7).

The reading ability expressed in descriptor 2 shows that the student needs to realize that the text consists of interconnected parts, forming a network of meaning, and identify the elements that promote the chaining of the text, which can be done using pronouns, synonymy relations or related words. In descriptor 12, it is necessary that the student is able to identify the logical-discursive relations present in the text, marked by the semantic relations of different natures, such as relations of comparison, of concession, of addition. Being able to establish the semantic relationship between the parts of the text is fundamental in

the construction of the network of meaning of the text and, consequently, in the identification of the corresponding mathematical proposition and its relations.

The skills of descriptors 2 and 12 can be identified in question 17 of the Mathematics test expressed in figure 7.

The mayor of the city of Belomar did a survey with the tourists that arrived at the city. The question was: “What means of transportation did you use to get to Belomar?”  
The table below shows the tourists’ responses:

Meio de transporte utilizado	Número de pessoas
Avião	128
Carro	450
Ônibus	589
Trem	90

[T.N.]  
*Meio de transporte utilizado: means of transport*  
*Avião: airplane*  
*Ônibus: bus*  
*Trem: train*  
*Número de pessoas: number of people*

How many tourists, in total, answered the survey of the mayor of Belomar?  
(A) 1 039  
(B) 1 129  
(C) 1 167  
(D) 1 257

Inferences:  
I1- The mayor did a survey with the tourists arriving in Belomar.  
I2- The relation between the means of transport and the number of tourists interviewed is expressed in the table.  
I3- Add the number of tourists to get the answer to the question.

Propositions:  
P1 - The mayor of the city of Belomar did a survey with the tourists that arrived at the city.  
P2 - The table below shows the responses of the tourists.  
P3 - How many tourists, in total, answered the survey of the mayor of Belomar? (The question of the problem)

Source: Devolutivas Pedagógicas platform (own translation).

**Figure 7** – Question 17 of the standardized test of 6th year Mathematics

To solve the mathematical problem in figure 7, the student must establish a relation of meaning between the parts of the text. In this case, the relationship of meaning is

expressed between “the tourists”, written in the problem statement, and “number of people”, title of the table column that is part of the mathematical problem. Without making this semantic relationship between the parts of the text, the student cannot identify the number of tourists to perform the summation, which corresponds to the mathematical proposition, to arrive at the result requested in the question of the problem.

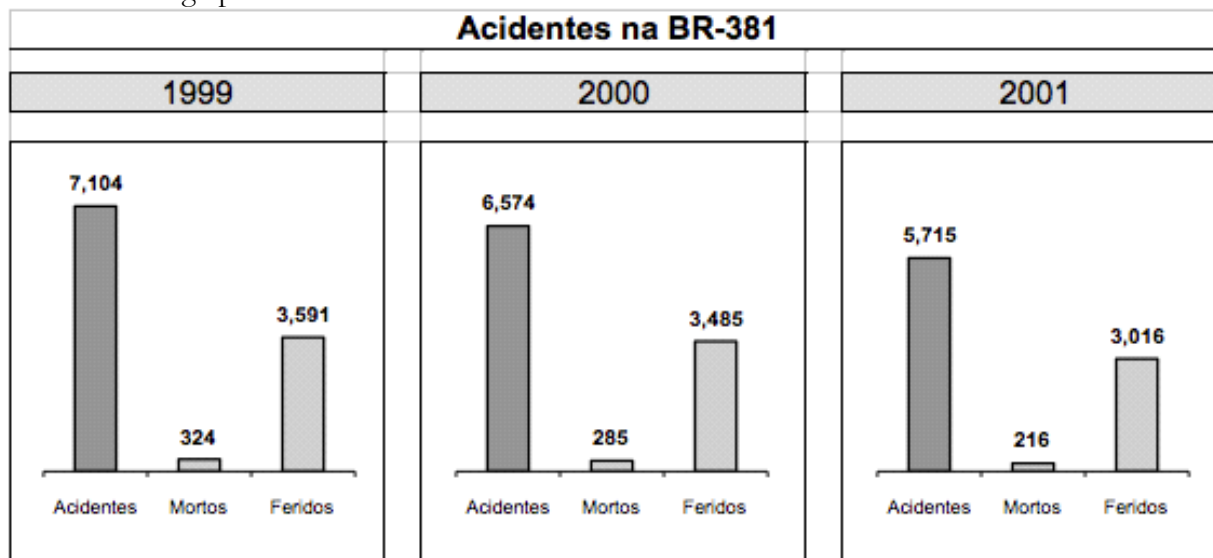
The student’s ability to establish relationships between parts of the text, identifying coherence in the logical-discursive relationship established between the text of the statement in which the mayor interviews tourists and the description of the table column by number of people (tourists), shows that they grasped the overall coherence between the parts of the text of the question.

In topic IV, descriptor 8, “Establish a cause/consequence relationship between parts and elements of the text”, it is understood as cause/consequence the relations between elements of the text in which one is the result of the other. With the ability to identify the motive that originated a fact presented in the text, the student is able to recognize, for example, relations of cause and effect, affirmation and confirmation, problem and solution, among others, and transform that linguistic information into mathematical proposition to solve the problem. The skills of descriptor 8 can be identified in question 25 of the standardized 9th grade Mathematics test, shown in figure 8 (next page).

Through descriptor 8 we can verify the student’s ability to recognize the relations expressed between the elements of the mathematical problem (bar graphs indicating the number of accidents per year) that are organized in the text, so that one result differs from the other (decreases the size of the chart column, consequently decreases the number of accidents per year). This graph reading ability leads the student to find the solution to the problem, establishing cause and effect relationships between the size of the graph columns, which represents the number of accidents per year, and the decrease in the number of accidents.

In topic V, the descriptors 18 and 19, respectively, “Recognize the effect of meaning arising from the choice of a particular word or expression” and “Recognize the effect of meaning resulting from the exploration of orthographic and/or morphosyntactic resources” pertain to the choice of a word in the text and the intentionality of that choice.

Observe these graphs:



SOURCE: Federal Highway Police

[T.N.]

*Acidentes: accidents;*

*Mortos: dead;*

*Feridos: injured*

According to the information in these graphs comparing the number of accidents and the number of injured in BR-381 from 1999 to 2001, it is understood that:

- (A) the number of accidents and the number of injured decreased.
- (B) the number of accidents decreased and the number of injured increased.
- (C) the number of accidents increased and the number of injured decreased.
- (D) the number of accidents and the number of injured increased.

Inferences:

My inferences would be these:

I1 - Compare columns of accidents and injured.

I2 - Compare the values in each column of accidents and injured.

I3 - Check that the values above the columns decrease from one year to another.

Propositions:

P1 - Comparison between the number of accidents and the number of injured.

P2 - Comparison of the number of accidents during the years 1999 to 2001.

P3 - Comparison of the number of injured during the years 1999 to 2001.

P4 - Was there a decrease in the number of accidents during these 3 years? (The question of the problem)


Source: Devolutivas Pedagógicas platform (own translation).


**Figure 8** – Question 25 of the standardized test of 9th year Mathematics


The selection of the word to be used in the construction of a text says a lot about its intentions as clues to find the meaning of the text. The choice of certain words or

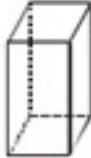
expressions as well as the use of figures, images and graphics should be processed by the reader as another clue to solve the problem. Descriptor 18 aims to verify if the reader is aware of the form and organization of all information in the text and to identify that these subtleties interfere in the construction of meaning to find the answer of the problem. The descriptors of topic V are mobilized in question 15 of the standardized test of the 6th year of Mathematics, figure 9.

The figure that has all faces square is:

(1)  


(2)  


(3)  


(4)  


(A) 1  
(B) 2  
(C) 3  
(D) 4

Inferences:  
I1- Identify the figures that have square faces.  
I2- Identify the figure that has ALL faces square.

Propositions:  
P1 - Identify the figures that have a square face.  
P2 - Identify the figure that has ALL faces square.  
P3 - The figure that has all faces square is the square.  
P4 - Which figure has all faces square? (The question of the problem)

Source: Devolutivas Pedagógicas platform (own translation).

### Figure 9 – Question 15 of the standardized test of 6th year Mathematics

The use of the quantifier “all” aims to check if the student knows which figure has all square faces. This quantifier (pronoun) was intentionally used to verify the effect of meaning that the descriptors 18 and 19 require the student to reflect on the form and the organization of the text and also if they can identify the marks used in the text that lead to the resolution of the problem.

Descriptor 14, from topic V, “Identifying the effect of meaning arising from the use of punctuation and other notations”, refers to the effect of meaning that the use of punctuation and other notation mechanisms (such as italics, bold type, uppercase, font size, etc.) can create in the text. Using these features requires the ability of the reader to identify effects of meaning that can be created in the text. In the wording of mathematical problems, these marks are not commonly used features.

In the part of the text that presents the question (of the problem) to be solved in the mathematical problem, the question mark (?) or even any graphic punctuation is used. In this regard, we can infer that there may be intentionality in this absence of interrogation or in the use of another graphic signal or even of not using any punctuation. Considering the possibility of an interference in the success of the solution of the problem, we performed a survey of the use of the graphic signal in the phrasing of questions of the Mathematics standardized test of the 6th and 9th years (table 1).

**Table 1** – Identifying the use of punctuation in questions of the Mathematics standardized test

Standardized test by year/grade	Question mark ?	Period .	Colon :	Did not use any punctuation marks
6th year	18	0	2	2
9th year	13	0	4	9
<b>Total</b>	31	0	6	11

Source: Formulated by the authors.

The Mathematics standardized tests of the 6th and 9th year of elementary school have 48 questions; of these, no question uses a period (punctuation) as the mark of the question of the mathematical problem; 31 questions use the question mark to determine the question of the problem; 11 do not use any punctuation marks and 6 questions use the colon as a punctuation mark of the statement of the question of the problem to be answered. The use of these punctuation features has an intentionality and an effect that the student needs to assign meaning to and identify that meaning in the context of the statement to achieve success in solving the problem.

#### FINAL CONSIDERATIONS

By establishing the relation between the processes of reading comprehension and the resolution of mathematical problems, we have identified how important proficiency in reading is for problem solving. The skilled reader who developed the processes of reading comprehension can perform well in the test of mathematical problem solving. However, the unsuccessful reader will not perform well in this type of test. This type of reader does not have automated decoding processes, signalling a failure in the initial learning of reading, although they are in the final years (6th and 9th year) of elementary school, since their

attention is focused on the decoding of the written word and the cognitive resources are not directed towards understanding the text.

Linking the theoretical construct of the reading comprehension model (PERFETTI; STAFURA, 2014) with that of problem solving (PÓLYA, 2006) made it possible to pinpoint that the processes of reading comprehension are basic for solving mathematical problems. Considering the correlation between the processes of reading comprehension and solving mathematical problems, we identified the processes of reading comprehension in descriptors of three topics of the Portuguese Language reference matrix of *Prova Brasil* present in the Mathematics questions of the standardized test and that are necessary to solve them.

In face of the results, we agree with the studies that suggest mathematical problems as a textual genre to be worked on in Portuguese and Mathematics classes, as well as the need to work on the linguistic structures in problematics by the two disciplines.

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