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Theoretical Analysis of Growth and Collapse of Spherical Cavities

Estudo Analítico do Colapso e Crescimento de Cavidades Esféricas

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ABSTRACT

In the present study, the analytical equations of conical functions are used to describe the growth and collapse of vapor and spherical cavities in liquids (cavitation). The equations describing the expansion of the spherical finite universe from the cosmological models are also applied to the study of growth and collapse of vapor and air cavities within liquids. The relative simple method used here has the advantage of prescinding numerical and/or computational simulations and other methods. Although the differences between the results of the adiabatic collapse and the isothermal collapse are very small, according to the literature. The hypothesis of the isothermal collapse of the bubble is used and justified based on the concept of characteristic time for heat transfers.Consequently, available theoretical and experimental data for isothermal collapse of such cavities are also used and, in the sequence, adjusted to polynomial equations. These were used to describe the radius as a function of time during the collapse and growth of the cavities. An auxiliary function was used here with the time variable, resulting in a linear function of it. The results are presented in graphs showing the bubble radius as a function of time, for collapse or growth of the cavity, or bubble.

keywords cavities, collapse, conical, growth, cavitation

RESUMO

No presente estudo, as equações das cônicas são utilizadas na descrição do colapso e crescimento de cavidades esféricas de ar e vapor em meios líquidos (cavitação). As equações do modelo de crescimento do universo (atualmente em expansão) esférico finito da cosmologia são também adaptadas ao estudo do colapso e crescimento de cavidades de ar e vapor em meios líquidos. O modelo estudado, por ser essencialmente analítico e relativamente simples, tem a vantagem de prescindir de simulações e outros métodos numéricos ou computacionais. A hipótese de colapso isotérmico é utilizada e justificada através do conceito de tempo característico para trocas de calor, ainda que as diferenças entre os resultados de colapso isotérmico e colapso adiabático sejam muito pequenas, de acordo com a literatura. Assim sendo, dados teóricos e experimentais disponíveis na literatura do colapso isotérmico destas cavidades são utilizados na obtenção de equações polinomiais para descrição do raio em função do tempo durante o colapso e crescimento das mesmas. Uma função auxiliar foi usada, juntamente com a variável tempo, resultando numa função linear do mesmo. Os resultados são apresentados na forma de gráficos do raio da bolha em função do tempo, seja para crescimento, seja para o colapso da cavidade, ou bolha.

palavras-chave cavidades, colapso, cônicas, crescimento, cavitação

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Introduction

The bubbles, named cavities, nucleate from early-present micro-bubbles of air in the liquid medium. The phenomenon is named "cavitation inception" (Hammitt, 1980). The growth process involves the action of surface forces, and is the result of the balance of the forces due to pressure, viscosity, and inertia (Bazanini et al., 2018). In the end, it is just the interplay of the static force at the bubble wall by the surface tension and pressure differences and those caused by physical properties.

Cavitation involves the entire activity of the bubbles, or cavities, from their formation to their collapse. The consequences of cavitation (remarkably erosion) are responsible for damages in metallic and non-metallic solid structures submerged in liquids (especially in water), resulting in additional costs for the industry.

Some recent applications of cavitation are: detection of bubbles in oil helps the prediction of transformers maintenance; mixing a system consisting of two immiscible liquids, since, in cavitation conditions, fine drops of one liquid can be forced into the other; cleaning with ultrasound systems; foaming in beer; aeration of lakes; in industry, since cavitation is an exergonic process, the bubbles have been used in water jets to increase their cutting power and then reduce the cutting time, and in dentistry works.

Experimental studies using the rotating disc device with the goal of studying cavitation pits were realized by Bazanini (2017). Some pressure field calculations originated from bubble collapse can be seen in (Bazanini et al., 1999), for incompressible liquids.

With the main objective of a better understanding of the phenomenon, an analogy will be made here, working with simple conical equations as well as those from cosmology.

Based on data available in the literature, analytical equations for the bubble radius as a function of time will be obtained during its collapse or growth. Conical equations will also be used here to describe the collapse or growth of a bubble (or spherical cavity), as well as regressions of those available data, with the goal of obtaining analytical expressions for the bubble radius.

With the main objective of a better understanding of the phenomenon, an analogy will be made here, working with simple conical equations as well as those from cosmology, since the existing equations for the bubble motion in its complete form, known as Rayleigh–Plesset equations, have no known analytical solution (Bistafa & Bazanini, 1997).

Materials and methods

A first trial to the problem of a cavity growth or collapse (and the appearance of singularities) is to make an analogy between the bubble growth and the spherical finite universe model from cosmology (Novello, 2023), since both are in an expansion process. Then we can use equation (1) adjusted from Acosta et al. (1975):

$$R = 4.6t^{2/3}. \tag{1}$$

Equation (1) can be used to obtain the bubble radius as a function of time. Although singularities are mathematical subjects, they are not well accepted or explained in engineering and applied physics. In fact, Lifshitz and Khalatnikov showed that a general solution of the field equations of gravitation should not contain singularities (Landau & Lifshitz, 1980), these singularities becoming more mathematical entities, than physical ones. Anyway, for the bubble collapse (or growth), some hypotheses may be used in the calculations, such as isothermal or adiabatic, for example (Bazanini & Bressan, 2017).

Here, we chose for the isothermal hypothesis, based on calculations for water, as justified below, although the differences between the results of the adiabatic collapse and the isothermal collapse are very small, as can be seen in Bazanini and Bressan (2017), since the process is very fast.

Some measurements and calculations for the radius of the bubble as a function of time have been performed in the last few years by many researchers. Some of them are cited here, whose results are also used to base the conical model.

Muller et al. (2012) generated a single cavity by an electric spark. The temporal evolution of the cavity radius was recorded by a high-speed camera of 10,000 frames per second. Dular and Delgosha (2012) used two high-speed cameras at 3,840 frames per second. Zhang et al. (2017) also used a high-speed camera of

3,000 frames per second. Sinibaldi et al. (2018) made an experimental investigation of a single cavity laser induced with acquisitions at 160,000 frames per second. Laser-induced cavitation bubbles have also been studied recently (Zhong et al., 2021).

Justification of the isothermal hypothesis

Supponen et al. (2018) used an adimensionalized form of the radius and the time, equation (2), similar to the one used by Bazanini and Hoays (2008):

$$T = \frac{t}{R_0}\sqrt{\frac{P_\infty}{\rho_L}}. \tag{2}$$

The isothermal collapse of the bubble is considered based on the characteristic time of heat transfer. In the section Results, they are presented in graphs in the form of the radius of the bubble as a function of time, for the collapse or growth of the cavity (or bubble), in comparison with some results available in the references, together with the analytical equations of conical functions.

According to Dular and Delgosha (2012), for the process of growth and collapse of a cavity to be considered adiabatic (the bubble evolution is mainly driven by expansion and compression of gases) or isothermal (in this case, evaporation and condensation of water are the main driving mechanisms), the bubble life must be compared to the characteristic time of heat transfer, for each case. It is considered adiabatic if the bubble life is much shorter than the characteristic time, i.e., the process is too fast for heat transfer to occur. Otherwise, the process is considered isothermal. The characteristic time of heat transfer (Frank & Michel, 2005), given by equation (3), is:

$$\Delta t = \frac{(\rho_g C_{v_b} R)^2}{9\lambda_l \rho_l C_{p_l}}, \tag{3}$$

where ρ_g and ρ_l are the gas (or vapor) and liquid densities, respectively. R is the bubble radius. C_{v_g} and C_{p_l} are the specific heats of the gas and liquid, respectively. λ_l is the thermal conductivity of the liquid.

Therefore, the phenomenon will be treated as isothermal in the following calculations.

Data used for model validation

As justified in the previous discussion, the phenomenon of collapse will be considered isothermal, and theoretical data from Bistafa and Bazanini (1997) and experimental data from Bazanini and Bressan (2017) will be used, as listed in Table 1. It is possible to see that these values are very close to each other for each case. For water, with the bubble radius used in the present calculations (3.50 mm), obtained from experiments reported by Hammitt (1980), the characteristic time results in a value of 3.35×10^{-4} μs, far shorter than the bubble life, which is approximately 0.7 ms.

Table 1 - Cavity radius as a function of time for the cavity collapse.

Radius (mm)	Time (ms)	
	Experimental	Theoretical Isothermal
3.50	0.00	0.00
3.40	0.18	0.18
3.20	0.30	0.30
2.90	0.40	0.40
2.50	0.55	0.50
2.00	0.62	0.58
1.00	0.69	0.63
0.50	0.70	0.65

Theoretical isothermal data from Table 1 were used with the aim to calculate the radius as a function of time, in the polynomial form, resulting in equation (4):

$$R = -30.376t^3 + 18.921t^2 - 3.8904t + 3.533, \quad (4)$$

a little more precision is possible by including more terms, resulting in equation (5):

$$R = -605.97t^6 + 711.06t^5 - 185.34t^4 - 58.881t^3 + 27.037t^2 - 3.0726t + 3.5001. \quad (5)$$

The same procedure was applied to the experimental data from Table 1, resulting in equation (6):

$$R = 1.10245t^4 - 6.16327t^3 - 1.03998t^2 - 0.298213t + 3.53815 \quad (6)$$

Next, the auxiliary function $\chi(h, V)$ is calculated, as defined in Landau and Lifshitz (1959), that is, in fact, energy $\times$ time per unit of mass:

$$\chi = \phi - rV + t(h + \frac{1}{2}V^2), \quad (7)$$

where ϕ is the velocity potential, r is the radius of the cavity, t is the time, h is the enthalpy, and V is the velocity. Here, the velocity is taken as the collapse velocity of the cavity calculated by Bistafa and Bazanini (1997).

According to Chorlton (1967):

$$\phi = -rV \ln V. \quad (8)$$

Substituting equation (8) into equation (7), results in:

$$\chi = -rV(1 + \ln V) + t(h + \frac{1}{2}V^2). \quad (9)$$

Since the first term on the right-hand side of equation (9) is negligible when compared to the second one, equation (10) will be used:

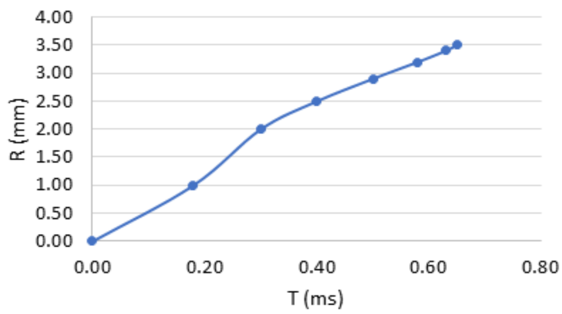
$$\chi = t(h + \frac{1}{2}V^2) \quad (10)$$

So, for water, χ can be expressed as a function of time and, consequently, as a function of the cavity radius.

Results and discussion

Equation (1), adapted from cosmology, was used to calculate the bubble radius as a function of time for a bubble growth, resulting in Figure 1.

Figure 1 - Radius as a function of time using equation (1).



Since the polynomial equations showed good results, and observing the shape of the radius versus time curves (Bazanini & Bressan, 2017), the use of conical equations is considered. The equation for a parabola may be used in the following form (Venturi, 2003):

$$(x - x_0)^2 = -C(y - y_0), \tag{11}$$

where, in the present calculations, $x = t$ and $y = R$.

For the cavity growth, by trial and error, we obtain:

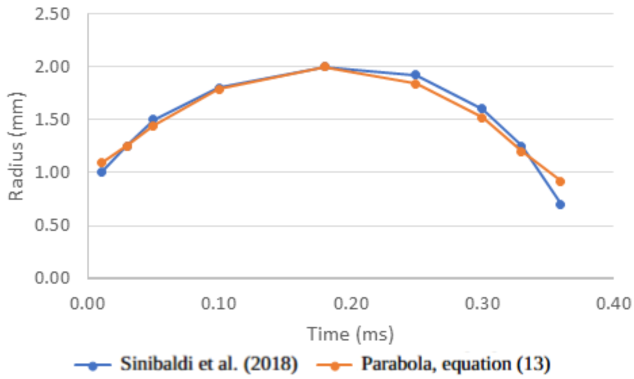
$$(t - 0.77)^2 = -0.14(R - 3.5). \tag{12}$$

We shall now apply the same procedure to data from Dular and Delgosha (2012), Muller et al. (2012), and Sinibaldi et al. (2018), and Zhang et al. (2017), establishing the constants of the conical equations for each case, since the purpose is to use a simple conical equation for every existing situation of growth and collapse, regardless of the circumstances around the bubble. So, we will have a conical equation with different constants for each set of data, of each reference, and so on.

In Figure 2, data from Sinibaldi et al. (2018) are compared to the ones obtained from the resulting parabola, equation (13), for the growth and collapse of the bubble, given by:

$$(t - 0.18)^2 = -0.03(R - 2). \tag{13}$$

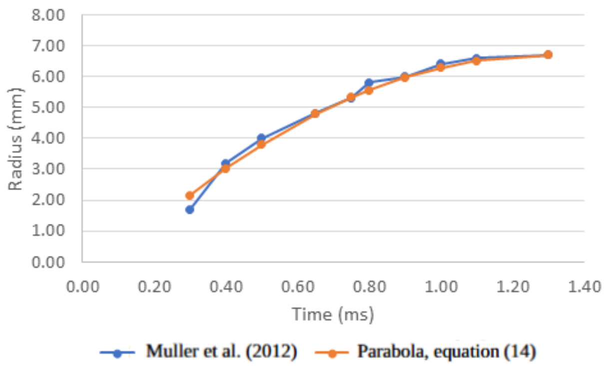
Figure 2 - Radius as a function of time using equation Radius versus time for collapse and growing of the cavity. Sinibaldi et al. (2018) x parabola, equation (13).



In Figure 3, data from Muller et al. (2012) are compared to the ones obtained for the resulting parabola, equation (14) , for the growth and collapse, resulting in the conical:

$$(t - 1.3)^2 = -0.22(R - 6.7). \tag{14}$$

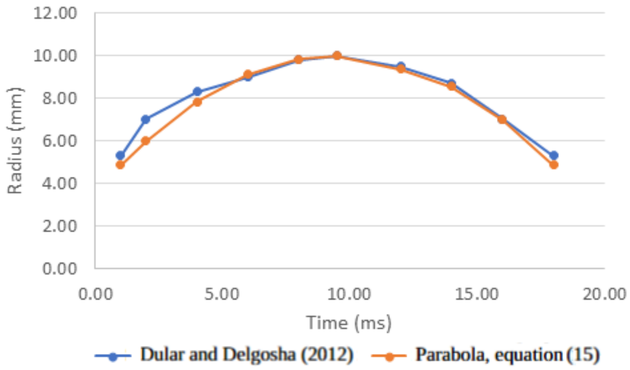
Figure 3 - Radius versus time for collapse growing of the cavity. Muller et al. (2012) x parabola, equation (14).



Next, in Figure 4, the same comparisons are made for data from Dular and Delgosha (2012). Now, the resulting conical equation is:

$$(t - 9.5)^2 = -14(R - 10). \tag{15}$$

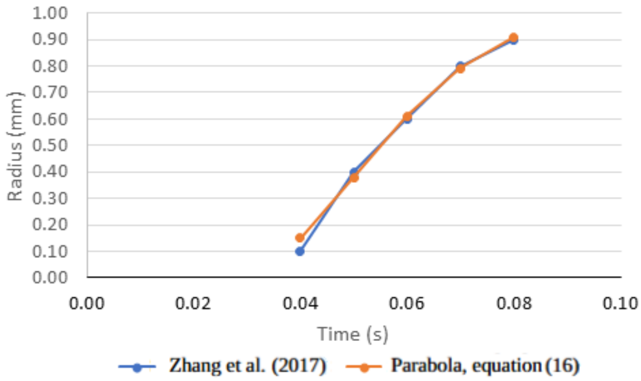
Figure 4 - Radius versus time for growing and collapse of the cavity. Dular and Delgosha (2012) x parabola, equation (15).



Finally, Figure 5 presents the comparative results between the data from Zhang et al. (2017) and the conic described in equation (16), given by:

$$(t - 0.29)^2 = -0.12(R - 2.3). \tag{16}$$

Figure 5 - Radius versus time for growing of the cavity. Zhang et al. (2017) x parabola equation (16).



Once again, good results were obtained for all cases for the conical model as well, except in the vicinities of the singularities (for radius zero, for instance), as previously explained.

Since the process is considered isothermal, the enthalpy h will be taken as a constant for water at ambient temperature of 20°C, thus $h = 2538$ kJ/kg (Bistafa & Bazanini, 1997).

Working with data from Table 1 and equation (10), Figures 6 and 7 were created for χ as a function of time t and the cavity radius $r = R$, for the cavity collapse, respectively. The velocity values V used in equation (10) were obtained from Bistafa and Bazanini (1997).

Although bubbles usually appear in clouds and the model used here is appropriate for one single bubble, it is important for a better understanding of the process of growth and collapse themselves, as well as the formation of the damage mechanisms on solid surfaces by the bubble, such as shock waves and micro-jets (Bazanini et al., 2017).

Figure 6 - Auxiliary function χ as a function of time.

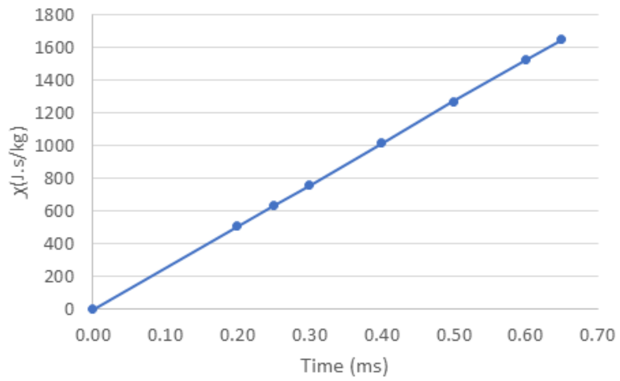
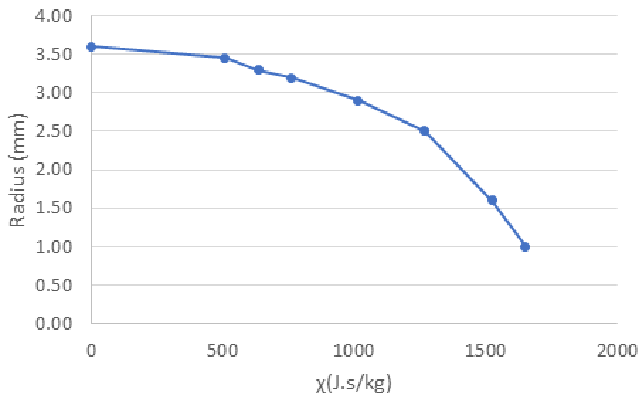


Figure 7 - Cavity radius R as a function of the auxiliary function χ



Conclusions

First of all, our test model used in the materials and methods for a spherical finite universe model from cosmology, in analogy with the cavitation bubble, showed good results, since both are submitted to governing equations of spherical expanding objects.

The parabolic behavior for the radius versus time curves worked remarkably well for both the growth and the collapse of the cavity, except in the vicinities of the singularity points.

As could be seen (as expected and already explained in the previous section), results become more reliable far from the singularity points, that is, those of zero radius.

The auxiliary function used here worked well in linearizing the time, resulting in a linear function of time.

Polynomial fitting using data available in the literature showed good results as well. Such equations, together with the conical equations, proved to be of very practical use, since the differential equations governing such phenomena have no analytical solutions, which require the development and testing of computational codes.

The equations obtained here showed an easiest way to apply to study of growth and collapse of a cavitation bubble.

Author contributions

G. Bazanini participated in: conceptualization, data curation, formal analysis, investigation, methodology, writing – original draft. R. K. Unfer participated in: supervision, validation, visualization, writing – revision and editing.

Conflicts of interest

No potential conflict of interest was reported by the authors.

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