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Robust Stabilization of $\mathscr{D}$-LQR-LMI Controllers by Norm-Bounded Uncertainties

Estabilização Robusta de Controladores $\mathscr{D}$-LQR-LMI por Incertezas Limitadas por Norma

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ABSTRACT

This study focuses on the synthesizing of Linear Quadratic Regulators (LQR) applied to linear systems with norm-bounded uncertainty models. Therefore, the main objective is to establish new LMI conditions to guarantee the robust $\mathscr{D}$-stability so that the transient response can be achieved through classical LMI restrictions. The mathematical analysis is based on the control design (by states feedback) of the power system stabilizer and the flexible AC transmission systems (FACTS) controllers, where these devices are able to operate under actuator faults. Finally, comparative analyses will be conducted with the literature considering factors such as the practical applicability and the transient response of signals of interest, including the evaluation of the controller behavior in two scenarios, *i.e.*, with fully operational actuators and under partial failures. Moreover, the LMI structure of the proposed theorem offers greater design flexibility, especially for high values of the parameters of the disk $D(q,r)$.

keywords LQR, LMIs, robust $\mathscr{D}$-stability, norm-bounded uncertainties

RESUMO

O presente trabalho enfatiza a síntese de Reguladores Lineares Quadráticos (LQR) para sistemas lineares incertos via modelos de incertezas limitadas por norma. Logo, objetiva-se obter novas condições LMIs que garantam a $\mathscr{D}$-estabilidade robusta, tal que a resposta transitória é alcançada através do uso de restrições clássicas de LMIs. A análise numérica é baseada no projeto de controle (por realimentação de estados) do estabilizador de um sistema de potência e do controlador flexível de sistemas de transmissão CA, tal que estes sejam hábeis a operar sob falhas nos atuadores.Por fim, as análises comparativas com a literatura serão realizadas considerando fatores como a aplicabilidade prática e a resposta transitória de sinais de interesse, incluindo a avaliação do comportamento dos controladores em dois cenários *i.e.*, com atuadores totalmente operacionais e sob falhas parciais. Ademais, a estrutura LMI do teorema proposto oferece maior flexibilidade de projeto, especialmente para valores elevados dos parâmetros do disco $D(q,r)$

palavras-chave LQR, LMIs, $\mathscr{D}$-estabilidade robusta, incertezas limitadas por norma

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Introduction

In the scope of Electrical Power Systems (EPS), certain dysfunctions in the charge, and even on long-distance power transmissions, can be responsible for causing low frequency oscillations, which can compromise the stability of a system if the dumping is not adequate (Bernardo et al., 2017; Rezende et al., 2021). To provide better damping for the generating units, Power System Stabilizers (PSS) are added (Sauer et al., 2017; Shayeghi et al., 2010; Soliman et al., 2011). However, the sole use of the PSS itself is not enough to guarantee the EPS stability, especially the oscillations that occur in multi-machine systems. Thus, Flexible AC Transmission Systems (FACTS) are used (Mohanty & Barik, 2011; Paserba, 2004). The project of these controllers must provide not only system stability, but also an adequate transitory response. This behavior can be obtained through the use of state or output feedback techniques, in which a robust response that is tolerant to failures is desirable (Soliman et al., 2011).

A common project method refers to the application of optimal control, and the theory developed by Lyapunov to guarantee asymptotic EPS stability under a closed loop. An optimal controller that employs restrictions in variables of state and control through the use of quadratic functions defines a problem known by a Linear Quadratic Regulator (LQR) (Bimarta & Kim, 2020; Caun et al., 2018, 2021; Xia et al., 2020). This project technique requires a solution to an algebraic equation by Riccati, which is a matrix. It is important to note that it is advantageous to combine LQR control with the Linear Matrix Inequalities (LMIs), because it allows treating concerns such as robustness, a requirement that might not even be considered in classic LQR theory (Olalla et al., 2009).

In this study, Chilali and Gahinet (1996) Chilali and Gahinet (1996) described the restrictions that guarantee the desirable region of transitory development in the complex plan of eigenvalues. In the scientific literature, it is possible to catalog a few propositions of $\mathscr{D}$-LQR-LMI control:

(a) Moheimani and Petersen (1996) proposed a robust controllers project, based on state feedback under the influence of norm-bounded uncertainties of the plant. The controller is designed to allocate poles of the closed loop mesh in a circular region, where the optimization problem considers the restrictions of Linear Matrix Equalities (LME) kind. Other particularities of the work include the specifications of the disc in the matrices of the model, by variables of state and uncertainty of the plant, even with the function cost being based on mathematical expected value;

(b) the work of Soliman et al. (2011) is different from Moheimani and Petersen (1996) because it considers a polytope of a model of uncertainties in the plant, the $\mathscr{D}$-stability set of restrictions are included in the process of optimization in the LMI form, and now the function cost is based in the output energy of the LQR problem;

(c) the Rezende et al. (2021) formulation can be cited because it adopts $\mathscr{D}$-stability restrictions in the form of additional LMIs. Therefore, it represents a direct application of the results of Chilali and Gahinet (1996).

Even in a project to control dynamic systems with an elevated number of state variables, the strategy for parametrization of weighting matrices $\mathscr{Q}$ and $\mathscr{R}$ becomes less practical. Moreover, when opting for the incorporation of restrictions in the eigenvalues of the local models of the uncertain system in a closed loop in the original formulation of the LQR-LMI control, Soliman et al. (2011) proposition creates a minor computational effort, which is advantageous on the extent that uncertainties are considered in the mathematical model of the system. In both cases, the pair $(\mathscr{Q}, \mathscr{R})$ are defined to obtain the feasibility of LMIs, and by this way it transfers the restrictions of Chilali and Gahinet (1996) to the function of guaranteeing the transitory performance of the system.

Therefore, it is important to note that guaranteeing the performance of robust dynamic systems in the event of uncertainties is essential for the reliability of advanced applications, making $\mathscr{D}$-stability a strategic tool to control complex and uncertain systems. Nowadays, academic literature has indicated a few different directions, such as the following:

1. use of $\mathscr{D}$-stability to obtain robust observers in Takagi-Sugeno (TS) systems, with variables of a non-measurable premise, improving the flexibility and robustness of the stability conditions (Ouhib & Kara, 2023, 2024);

2. analysis methods of robust $\mathscr{D}$-stability in systems with fractional order, using LMI restrictions of the allocation of eigenvalues to the problem of optimizing under the influence of parametric norm-bounded uncertainties (Ghorbani et al., 2023; Wei et al., 2023);

3. application of techniques based on the fuzzy-basis-dependent Lyapunov functional, for stochastic fuzzy systems, allowing less conservative stability conditions under the influence of delayed time variables and norm-bounded uncertainties (Zheng et al., 2023);

4. PID controllers for systems with norm-bounded uncertainties by time variants, employing LMIs with restrictions of *rank*, offering efficient solutions and robust performance for multivariable systems (Pradhan & Ghosh, 2022).

These approaches indicate the fundamental role of LMIs in the analysis and control of robust systems, especially when dealing with uncertainties and providing transient performance guarantees, as evidenced by the references discussed.

By what was said and by the knowledge of the authors, an analysis that contemplates the model of the plant/process under the point of view of norm-bounded uncertainties in the form of LMIs is not well discussed in the study of Moheimani and Petersen (1996). Therefore, the contributions of this work can be summarized as follows:

1. propose an LMI formulation of the LQR problem based on norm-bounded uncertainties;

2. explore the feasibility regions, computational cost and effort of the control signals in $\mathscr{D}$-LQR-LMI controller designs.

Throughout this paper, a matrix $\boldsymbol{P} \in \mathbb{R}^{n\times n}$ represented by $\boldsymbol{P}^T$ denotes a transpose, while $\boldsymbol{P}^{-1}$ is an inverse. If $\boldsymbol{P} > 0$, then it indicates that the matrix is positive definite.

To $\boldsymbol{x}(t) \in \mathbb{R}^{n\times 1}$, the notation $\dot{\boldsymbol{x}}(t)$ denotes its derivative. Moreover, it is considered $\boldsymbol{I}$ as the identity matrix of the appropriate dimension and '$*$' the therms of the LMIs inferred by symmetry.

Materials and methods

The uncertainty models are inserted in the analysis of a Linear Time Invariant system (LTI) usually formulated by equation (1):

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{1}$$

with matrices $\boldsymbol{A} \in \mathbb{R}^{n_x\times n_x}$, $\boldsymbol{B} \in \mathbb{R}^{n_x\times n_u}$ and the initial condition vector $\boldsymbol{x}(0) \in \mathbb{R}^{n_x}$ known. For an uncertain system, the polytopic model of uncertainties is represented by the equation (2):

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\alpha)\,\boldsymbol{x}(t) + \boldsymbol{B}(\alpha)\,\boldsymbol{u}(t),\ \boldsymbol{x}(0) = \boldsymbol{x}_0, \tag{2}$$

where matrices $\boldsymbol{A}(\alpha) \in \mathbb{R}^{n_x\times n_x}$ and $\boldsymbol{B}(\alpha) \in \mathbb{R}^{n_x\times n_u}$ are assumed to depend on the parameters α with $\alpha \in \mathbb{R}^N : \sum_{i=1}^{N} \alpha_i = 1,\ \alpha_i \geq 0$.

For the norm-limited uncertainties model applied to the plant or to the gain of the controllers, the mathematical convention for formulating LMIs is expressed as follows:

Lemma 1 For generic matrices, real, and known, $\boldsymbol{Z}$, $\boldsymbol{M}$, $\boldsymbol{N}$ of appropriate dimensions,

$$\boldsymbol{Z} + \boldsymbol{M}\boldsymbol{\Delta}\boldsymbol{N} + \boldsymbol{N}^T\boldsymbol{\Delta}^T\boldsymbol{M}^T < \boldsymbol{0},$$

such that $\|\boldsymbol{\Delta}\| < 1$, if and only if $\exists\, \varepsilon > 0$ which satisfies,

$$\boldsymbol{Y} + \varepsilon\boldsymbol{M}\boldsymbol{M}^T + \varepsilon^{-1}\boldsymbol{N}^T\boldsymbol{N} < \boldsymbol{0}.$$

Proof: Refer to Xie et al. (1991) for more details on the development.

Consider the following design conditions proposed by Soliman et al. (2011);

1. the eigenvalues of the closed loop must satisfy a minimum transitory development that involves the dumping factor of $\xi \in [0, 10, 0, 25]$, in a way to avoid the weary of the generator shaft and the time of stabilization ($t_e \in [10, 15]s$);

2. guarantee the energy minimization of the state variables and control signals of the LQR equation, given by equation (3):

$$\mathscr{J}_{\infty} = \min \int_0^{\infty} [\boldsymbol{x}(t)^T \mathscr{Q} \boldsymbol{x}(t) + \boldsymbol{u}(t)^T \mathscr{R} \boldsymbol{u}(t)] dt, \tag{3}$$

such that equation (3) establishes a mediated weighting by the positive matrices defined by $\mathscr{Q}$ and $\mathscr{R}$, that allow prioritizing the energy optimization of a state variable x_i (or a control sign of u_j, respectively) in relation to the other states (or control signs) of the system (Kumar et al., 2016).

The main conditions of $\mathscr{D}$-LQR-LMI available in the literature will be reproduced as follows.

Theorem 1 *Consider the system given in equation* (2)*, with the matrix pairs* $(\boldsymbol{A}, \boldsymbol{B}(\alpha))$ *and the guaranteed cost function by equation* (3)*. If a state feedback matrix* $\boldsymbol{F}$ *exists, and also matrices* $\boldsymbol{Y} = \boldsymbol{Y}^T > \boldsymbol{0}$ *and* $\boldsymbol{S}$ *such that the LMI given by equation* (4)*:*

$$\begin{bmatrix} -r^2\boldsymbol{Y} & * & * & * \\ \boldsymbol{AY} + q\boldsymbol{Y} + \boldsymbol{B_i S} & -\boldsymbol{Y} & * & * \\ \boldsymbol{Y}^T & \boldsymbol{0} & -\mathscr{Q}^{-1} & * \\ \boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & -\mathscr{R}^{-1} \end{bmatrix} < \boldsymbol{0}, \ \forall i, \tag{4}$$

is satisfied by all possible actuator failures. Therefore, the controller guarantees stabilization under a restriction of $D(q, r)$ *eigenvalues, and its gain is given by:*

$$\boldsymbol{F} = \boldsymbol{S}\boldsymbol{Y}^{-1},$$

and the guaranteed cost function has a superior limit of $J^* = q\boldsymbol{x}_0^T \boldsymbol{Y}^{-1} \boldsymbol{x}_0$.

Proof. Refer to Soliman et al. (2011) for more details.

Theorem 2 *Consider the system given in equation* (1) *and the guaranteed cost function given in equation* (3)*. If matrices* $\boldsymbol{Y} = \boldsymbol{Y}^T > \boldsymbol{0}$ *and* $\mathbf{S}$ *exists such that the LMIs, given in* (5)-(8)*, are satisfied:*

$$\min_{\boldsymbol{Y}=\boldsymbol{Y}^T, \boldsymbol{S}} Tr\{\boldsymbol{Y}\},$$

$$\begin{bmatrix} -(\boldsymbol{AY} - \boldsymbol{BS})^T - (\boldsymbol{AY} - \boldsymbol{BS}) & * & * \\ \boldsymbol{Y} & \mathscr{Q}^{-1} & * \\ \boldsymbol{S} & \boldsymbol{0} & \mathscr{R}^{-1} \end{bmatrix} \geq \boldsymbol{0}, \tag{5}$$

$$-2h_1\boldsymbol{Y} - (\boldsymbol{AY} - \boldsymbol{BS})^T - (\boldsymbol{AY} - \boldsymbol{BS}) \leq \boldsymbol{0}, \tag{6}$$

$$2h_2\boldsymbol{Y} + (\boldsymbol{AY} - \boldsymbol{BS})^T + (\boldsymbol{AY} - \boldsymbol{BS}) \leq \boldsymbol{0}, \tag{7}$$

$$\begin{bmatrix} sen(\theta)\left((\boldsymbol{AY}-\boldsymbol{BS})^T+(\boldsymbol{AY}-\boldsymbol{BS})\right) & -cos(\theta)\left((\boldsymbol{AY}-\boldsymbol{BS})^T-(\boldsymbol{AY}-\boldsymbol{BS})\right) \\ cos(\theta)\left((\boldsymbol{AY}-\boldsymbol{BS})^T-(\boldsymbol{AY}-\boldsymbol{BS})\right) & sen(\theta)\left((\boldsymbol{AY}-\boldsymbol{BS})^T+(\boldsymbol{AY}-\boldsymbol{BS})\right) \end{bmatrix} \leq \boldsymbol{0}. \quad (8)$$

Therefore, the state feedback gain

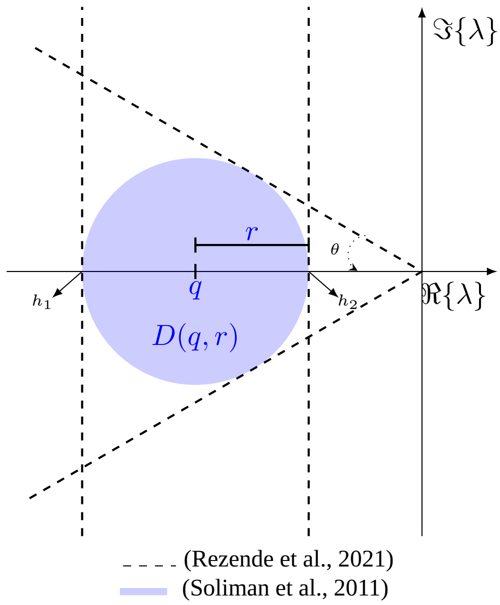
$$\boldsymbol{F} = \boldsymbol{SY}^{-1},$$

guarantees the quadratic $\mathscr{D}$-stability of the system given by equation (1).

Proof. Refer to Rezende et al. (2021) for more details.

In Theorems 1 and 2, $\mathscr{D}$-stability restrictions are introduced in distinct forms. Theorem 1 adopts a region defined by $D(q,r)$, which represents the center disc of q and radius of r, and Theorem 2 considers the classic approach described by Chilali and Gahinet (1996), although the specifications of performance are equivalent, see Figure 1.

Figure 1 - LMI restrictions of specification regions.



In comparatively terms, the formulation of Theorem 1 proposes the incorporation of LMI restrictions of region $D(q,r)$ in the original LQR-LMI problem. Therefore, it promotes the reduction of the computational cost in relation to Theorem 2, even if it is a more restrictive set. In addition, Theorem 1 represents a more general formulation because the LMI conditions guarantee robust stabilization. A deficiency in the conditions of Theorem 1 refers to the use of an uncertainty model in which its computational complexity grows exponentially in relation to the number of uncertainties existent in the plant/process (Boyd et al., 1994).

The $\mathscr{D}$-LQR problem based on LMI: modeling by norm-bounded uncertainties

Consider the continuous time linear system with norm-bounded uncertainties as represented by equation (9):

$$\dot{\boldsymbol{x}}(t) = (\boldsymbol{A}+\Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B}+\Delta\boldsymbol{B})\boldsymbol{u}(t), \quad (9)$$

in which $\Delta\boldsymbol{A} = \boldsymbol{D\Lambda E_1}$ and $\Delta\boldsymbol{B} = \boldsymbol{D\Lambda E_2}$, for $\|\boldsymbol{\Lambda}\| < 1$. Therefore, Theorem 3 provides necessary and sufficient conditions for quadratic stabilization based on the Riccati equation.

Theorem 3 *Consider the system presented in equation* (9), *with cost function defined by equation* (3). *Hence, the optimal cost-value limit* $\mathscr{J}_\infty^*$ *is given by*

$$\mathscr{J}_\infty^* = inf\left\{tr\left(\frac{\alpha}{r^2}\boldsymbol{P}^+\right) : \boldsymbol{P}^+ > \boldsymbol{0},\ \epsilon > 0\right\},$$

with the pair $(\boldsymbol{P}^+, \epsilon)$ *satisfying the expressions given in equations* (10) *and* (11)*:*

$$(\boldsymbol{P}^+)^{-1} - \epsilon \boldsymbol{D_r}\boldsymbol{D_r}^T > \boldsymbol{0}, \tag{10}$$

$$\left\{\left(\boldsymbol{A_r} - \boldsymbol{B_r}(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1}\boldsymbol{E_{2r}}^T\boldsymbol{E_{1r}}\right)^T \times \left(\boldsymbol{P}^{-1} - \epsilon\boldsymbol{D_r}\boldsymbol{D_r}^T + \epsilon\boldsymbol{B}(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1}\boldsymbol{B_r}^T\right)^{-1} \times \left(\boldsymbol{A_r} - \boldsymbol{B_r}(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1}\boldsymbol{E_{2r}}^T\boldsymbol{E_{1r}}\right)\right\} + \frac{1}{\epsilon}\boldsymbol{E_{1r}}^T\left(\boldsymbol{I} - \boldsymbol{E_{2r}}(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1}\boldsymbol{E_{2r}}^T\right)\boldsymbol{E_{1r}} - \boldsymbol{P} + \mathscr{Q} = 0. \tag{11}$$

Consequently, the control law given by equation (12)

$$\boldsymbol{u}(t) = -(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1} \times \left\{\epsilon\boldsymbol{B_r}^T\left(\boldsymbol{P}^{-1} - \epsilon\boldsymbol{D_r}\boldsymbol{D_r}^T + \epsilon\boldsymbol{B_r}(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1}\boldsymbol{B}^T\right)^{-1} \times \left(\boldsymbol{A_r} - \boldsymbol{B_r}(\epsilon\mathscr{R} + \boldsymbol{E_{2r}}^T\boldsymbol{E_{2r}})^{-1}\boldsymbol{E_{2r}}^T\boldsymbol{E_{1r}}\right) + \boldsymbol{E_{2r}}^T\boldsymbol{E_{1r}}\right\}\boldsymbol{x}(t), \tag{12}$$

ensures the D-quadratic guaranteed cost with the cost matrix $\tilde{\boldsymbol{P}} = \dfrac{\alpha}{r^2}\boldsymbol{P}^+$.

Proof Refer to Moheimani and Petersen (1996) for more details.

The novelty of this work lies in the formulation of the LMI conditions of Theorem 3 in the region of performance of Theorem 1, going from LTI systems described by norm-bounded uncertainties, as described in Theorem 4.

Theorem 4 *The system described in equation* (9) *is stable by* $\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{x}(t)$ *with the eigenvalues contained in the disc of radius* r *centered in* $-q$*, and guaranteed cost* $\mathscr{J}_\infty$ *inferior to* μ*, if matrices* $\boldsymbol{Z}$, $\boldsymbol{W} = \boldsymbol{W}^T > \boldsymbol{0}$ *and a scalar* $\varepsilon > 0$ *exists such that the LMIs in equations* (13) *and* (14) *are satisfied:*

$$\min_{\varepsilon,\ \boldsymbol{Z},\ \boldsymbol{W}} \mu$$

subject to,

$$\begin{bmatrix} \mu & * \\ \boldsymbol{x}(0) & \dfrac{\boldsymbol{W}}{q} \end{bmatrix} > \boldsymbol{0}, \tag{13}$$

$$\begin{bmatrix} -r^2\boldsymbol{W} & * & * & * & * & * \\ \boldsymbol{AW} + \boldsymbol{BZ} + q\boldsymbol{W} & -\boldsymbol{W} & * & * & * & * \\ \boldsymbol{W} & \boldsymbol{0} & -\mathscr{Q}^{-1} & * & * & * \\ \boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & -\mathscr{R}^{-1} & * & * \\ \boldsymbol{0} & \varepsilon\boldsymbol{D}^T & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon I & * \\ \boldsymbol{E_1}\boldsymbol{W} + \boldsymbol{E_2}\boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon I \end{bmatrix} < \boldsymbol{0}. \tag{14}$$

Therefore, the optimal feedback states gain $\boldsymbol{K} = \boldsymbol{Z}\boldsymbol{W}^{-1}$ *guarantee robust* $\mathscr{D}$*-stability of the system (9).*

Proof. Consider the system described by equation (9) and control law $\boldsymbol{u}(t) = -\boldsymbol{K}\boldsymbol{x}(t)$, which inequality is obtained by (15) (Moheimani & Petersen, 1996):

$$\begin{bmatrix} -r^2\boldsymbol{P} + \mathscr{Q} + \boldsymbol{K}^T\mathscr{R}\boldsymbol{K} & * \\ \boldsymbol{A}_{cl} + q\boldsymbol{I} + \boldsymbol{D}\boldsymbol{\Lambda}(\boldsymbol{E_1} - \boldsymbol{E_2}\boldsymbol{K}) & -\boldsymbol{P}^{-1} \end{bmatrix} < \boldsymbol{0}, \tag{15}$$

with $\boldsymbol{A}_{cl} = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$. Applying the congruence transformation by left multiplying equation (15) by $\begin{bmatrix} \boldsymbol{W} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}$ and right multiplying it by $\begin{bmatrix} \boldsymbol{W} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix}^T$ with variable changes $\boldsymbol{W} = \boldsymbol{P}^{-1}$, we obtain (16),

$$\begin{bmatrix} -r^2\boldsymbol{W} + \boldsymbol{W}\mathscr{Q}\boldsymbol{W} + \boldsymbol{W}\boldsymbol{K}^T\mathscr{R}\boldsymbol{K}\boldsymbol{W} & * \\ \boldsymbol{A}_{cl}\boldsymbol{W} + q\boldsymbol{W} + \boldsymbol{D}\boldsymbol{\Lambda}(\boldsymbol{E_1} - \boldsymbol{E_2}\boldsymbol{K})\boldsymbol{W} & -\boldsymbol{W} \end{bmatrix} < \boldsymbol{0}. \tag{16}$$

Subsequently, by applying the Schur's complement to the matrices pair $\{\mathscr{Q}, \mathscr{R}\}$ and making $\boldsymbol{Z} = -\boldsymbol{K}\boldsymbol{W}$, equation (17) is obtained:

$$\begin{bmatrix} -r^2\boldsymbol{W} & * & * & * \\ \boldsymbol{A}\boldsymbol{W} + \boldsymbol{B}\boldsymbol{Z} + q\boldsymbol{W} + \boldsymbol{D}\boldsymbol{\Lambda}\boldsymbol{E_1}\boldsymbol{W} + \boldsymbol{D}\boldsymbol{\Lambda}\boldsymbol{E_2}\boldsymbol{Z} & -\boldsymbol{W} & * & * \\ \boldsymbol{W} & \boldsymbol{0} & -\mathscr{Q}^{-1} & * \\ \boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & -\mathscr{R}^{-1} \end{bmatrix} < \boldsymbol{0}, \tag{17}$$

or, equivalently

$$\begin{bmatrix} -r^2\boldsymbol{W} & * & * & * \\ \boldsymbol{A}\boldsymbol{W} + \boldsymbol{B}\boldsymbol{Z} + q\boldsymbol{W} & -\boldsymbol{W} & * & * \\ \boldsymbol{W} & \boldsymbol{0} & -\mathscr{Q}^{-1} & * \\ \boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & -\mathscr{R}^{-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{D} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\Lambda} \begin{bmatrix} \boldsymbol{E_1}\boldsymbol{W} + \boldsymbol{E_2}\boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{W}\boldsymbol{E_1}^T + \boldsymbol{Z}^T\boldsymbol{E_2}^T \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\Lambda} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^T & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} < \boldsymbol{0}. \tag{18}$$

From (18), *Lemma 1* is applied, resulting in equation (19):

$$\begin{bmatrix} -r^2\boldsymbol{W} & * & * & * \\ \boldsymbol{A}\boldsymbol{W} + \boldsymbol{B}\boldsymbol{Z} + q\boldsymbol{W} & -\boldsymbol{W} & * & * \\ \boldsymbol{W} & \boldsymbol{0} & -\mathscr{Q}^{-1} & * \\ \boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & -\mathscr{R}^{-1} \end{bmatrix} + \varepsilon \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{D} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^T & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} \boldsymbol{W}\boldsymbol{E_1}^T + \boldsymbol{Z}^T\boldsymbol{E_2}^T \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{E_1}\boldsymbol{W} + \boldsymbol{E_2}\boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} < \boldsymbol{0}. \tag{19}$$

Finally, by applying Schur's complement in (19) inequality, the (20) LMI is obtained as follows:

$$\begin{bmatrix} -r^2\boldsymbol{W} & * & * & * & * & * \\ \boldsymbol{AW}+\boldsymbol{BZ}+q\boldsymbol{W} & -\boldsymbol{W} & * & * & * & * \\ \boldsymbol{W} & \boldsymbol{0} & -\mathscr{Q}^{-1} & * & * & * \\ \boldsymbol{Z} & \boldsymbol{0} & \boldsymbol{0} & -\mathscr{R}^{-1} & * & * \\ \boldsymbol{0} & \varepsilon\boldsymbol{D}^T & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon I & * \\ \boldsymbol{E_1W}+\boldsymbol{E_2Z} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\varepsilon I \end{bmatrix} < \boldsymbol{0}. \tag{20}$$

To prove the sufficiency of the (13) inequality, the results of Soliman et al. (2011) will be considered. Therefore, we define $V(\boldsymbol{x}) = \boldsymbol{x}(t)^T\boldsymbol{Px}(t)$ as a Lyapunov function candidate, where $\boldsymbol{P} = \boldsymbol{P}^T > \boldsymbol{0}$. Reorganizing the inequality $-r^2\boldsymbol{P} + (\boldsymbol{A}_{cl} + q\boldsymbol{I})^T\boldsymbol{P}(\boldsymbol{A}_{cl} + q\boldsymbol{I}) < -\mathscr{Q} - \boldsymbol{K}^T\mathscr{R}\boldsymbol{K}$, (21) is obtained:

$$\boldsymbol{A}_{cl}^T\boldsymbol{P} + \boldsymbol{PA}_{cl} < -\frac{1}{q}\boldsymbol{A}_{cl}^T\boldsymbol{PA}_{cl}^T - \frac{q^2-r^2}{q}\boldsymbol{P} - (\frac{\mathscr{Q}+\boldsymbol{K}^T\mathscr{R}\boldsymbol{K}}{q}) < -(\frac{\mathscr{Q}+\boldsymbol{K}^T\mathscr{R}\boldsymbol{K}}{q}) < \boldsymbol{0}. \tag{21}$$

By left multiplying equation (21) by $\boldsymbol{x}(t)^T$ and right multiplying it by $\boldsymbol{x}(t)$, we have (22):

$$\boldsymbol{x}(t)^T(\boldsymbol{A}_{cl}^T\boldsymbol{P} + \boldsymbol{PA}_{cl})\boldsymbol{x}(t) < -\boldsymbol{x}(t)^T(\frac{\mathscr{Q}+\boldsymbol{K}^T\mathscr{R}\boldsymbol{K}}{q})\boldsymbol{x}(t). \tag{22}$$

Since $\dot{V}(\boldsymbol{x}) = \boldsymbol{x}(t)^T(\boldsymbol{A}_{cl}^T\boldsymbol{P}+\boldsymbol{PA}_{cl})\boldsymbol{x}(t)$, the cost function (19) is obtained by integrating expression (22) as follows:

$$\int_0^\infty \boldsymbol{x}(t)^T\left(\frac{\mathscr{Q}+\boldsymbol{K}^T\mathscr{R}\boldsymbol{K}}{q}\right)\boldsymbol{x}(t)dt < V(\boldsymbol{x})\Big|_0^\infty = \boldsymbol{x}'(0)\boldsymbol{Px}(0). \tag{23}$$

Finally, the LMI (24) form of the upper bound for $\mathscr{J}_\infty$ was obtained by Schur's complement:

$$\begin{bmatrix} \mu & * \\ \boldsymbol{x}(0) & \dfrac{\boldsymbol{W}}{q} \end{bmatrix} > \boldsymbol{0},\ \text{com } \boldsymbol{W} = \boldsymbol{P}^{-1}. \tag{24}$$

Thus, Theorem 4 is proved.

Results and discussion

In this section, the discussed theorems are applied to a practical electrical power systems problem, discussed by Soliman et al. (2011). The objective is to explain the advantage between the uncertainty models in terms of feasibility tests related to disc parameters $D(q,r)$, the time behavior of the state variables and the intensity level of the control signal. The solutions to the LMIs were obtained in a computational environment using MATLAB software through the LMIlab package.

System description

Consider the system illustrated in Figure 2, which is made by an infinite bus generator under the action of two PSS units

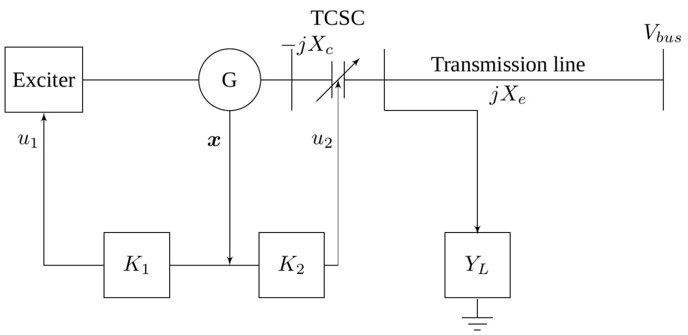
As described by Soliman et al. (2011), the $\boldsymbol{K}_1$ PSS is responsible for the action of the generator, while the $\boldsymbol{K}_2$ acts on the shooting angle of the thyristor, which means the control vector $u(t) \in \mathbb{R}^2$.

In this example, the state variables $\boldsymbol{x}(t) \in \mathbb{R}^4$ of the linearized incremental model around the operating point are defined as follows:

$$\boldsymbol{x}(t) = \begin{bmatrix}\Delta\delta & \Delta\omega & \Delta E_q' & \Delta E_f\end{bmatrix},$$

where δ is the torque angle, ω the angular velocity, E_q' axis voltage "q" of the transient reactance and E_f is the field voltage.

Figure 2 - Diagram of the infinity bus system of a unique machine controlled by a thyristor.



Adapted from "Guaranteed-cost reliable control with regional pole placement of a power system" by H. M. Soliman, A. Dabroum, M. S. Mahmoud and M. Soliman, 2011, Journal of the Franklin Institute, 348(5), 884–898.

In Soliman et al. (2011), the space equations of the state of the dynamic system described in equation (1) are obtained by a linear approximation, as the matrices of the linearized incremental model are given by:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0588 & 0 & -0.1303 & 0 \\ -0.09 & 0 & -0.1957 & 0.1289 \\ 95.532 & 0 & -815.93 & -20 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0.0704 \\ 0 & 0.0177 \\ 1000 & 93.846 \end{bmatrix}.$$

The objective is to project a robust controller ($\boldsymbol{K}$) with respect to the power losses in actuators. Considering a fault channel from the actuator to the controller, the circumstance of the actuators failing partially, completely, or not failing are illustrated. Thus, a polytope of four vertices can be formed as follows:

- Vertex 1 (100% of power in $\boldsymbol{K}_1$ and $\boldsymbol{K}_2$ PSS):

$$\boldsymbol{A_1} = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0588 & 0 & -0.1303 & 0 \\ -0.09 & 0 & -0.1957 & 0.1289 \\ 95.532 & 0 & -815.93 & -20 \end{bmatrix}, \quad \boldsymbol{B_1} = \begin{bmatrix} 0 & 0 \\ 0 & 0.0704 \\ 0 & 0.0177 \\ 1000 & 93.846 \end{bmatrix}.$$

- Vertex 2 (100% of power in $\boldsymbol{K}_1$ PSS and 60% in $\boldsymbol{K}_2$ PSS):

$$\boldsymbol{A_2} = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0588 & 0 & -0.1303 & 0 \\ -0.09 & 0 & -0.1957 & 0.1289 \\ 95.532 & 0 & -815.93 & -20 \end{bmatrix}, \quad \boldsymbol{B_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.0422 \\ 0 & 0.0106 \\ 1000 & 56.3076 \end{bmatrix}.$$

- Vertex 3 (60% of power in $\boldsymbol{K}_1$ PSS and 100% in $\boldsymbol{K}_2$ PSS):

$$\boldsymbol{A_3} = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0588 & 0 & -0.1303 & 0 \\ -0.09 & 0 & -0.1957 & 0.1289 \\ 95.532 & 0 & -815.93 & -20 \end{bmatrix}, \quad \boldsymbol{B_3} = \begin{bmatrix} 0 & 0 \\ 0 & 0.0704 \\ 0 & 0.0177 \\ 600 & 93.846 \end{bmatrix}.$$

- Vertex 4 (60% of power in $\boldsymbol{K}_1$ and $\boldsymbol{K}_1$ PSS's):

$$\boldsymbol{A_4} = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0588 & 0 & -0.1303 & 0 \\ -0.09 & 0 & -0.1957 & 0.1289 \\ 95.532 & 0 & -815.93 & -20 \end{bmatrix}, \quad \boldsymbol{B_4} = \begin{bmatrix} 0 & 0 \\ 0 & 0.0422 \\ 0 & 0.0106 \\ 600 & 56.3076 \end{bmatrix}.$$

However, the equivalent representation for the state spaces via norm-bounded uncertainties is described by equation (25):

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 377 & 0 & 0 \\ -0.0588 & 0 & -0.1303 & 0 \\ -0.09 & 0 & -0.1957 & 0.1289 \\ 95.532 & 0 & -815.93 & -20 \end{bmatrix} \boldsymbol{x}(t) + \left(\begin{bmatrix} 0 & 0 \\ 0 & 0.0563 \\ 0 & 0.0142 \\ 800 & 75.0768 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.0141 \\ 0 & 0.0035 \\ 200 & 18.7692 \end{bmatrix} \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \boldsymbol{u}(t). \tag{25}$$

Furthermore, the $D(q,r)$ region of the complex plan of the eigenvalues of the closed loop is specified adopting typical values in power applications, with the dumping $\xi \in [0,\ 1,\ 0,25]$ factor and setting time of the oscillations $t_s \in [10, 15]$s (Paserba, 2004). Therefore, the region that meets the desired specifications (Soliman et al., 2011) is described by equation (26):

$$D(q,r) = \{x + jy \in \mathbb{C} :\ (x+11)^2 + y^2 < 10,64^2\}. \tag{26}$$

This region differs from the one used in Theorem 2, in which to obtain the specifications of the proposed system it is necessary to comply with the following parameters $h_1 = 21.64$, $h_2 = 0.36$ e $\theta = 75.5°$.

Simulations

Admitting the $\mathscr{Q} = \boldsymbol{I}_4$, $\mathscr{R} = \boldsymbol{I}_2$ matrices and the initial conditions $\boldsymbol{x}(0) = [0, 1\ 0\ 0\ 0]$. The solutions to the optimization problems of Theorem 1 ($\boldsymbol{K}_{T_1}$), Theorem 2 ($\boldsymbol{K}_{T_2}$), and Theorem 4 ($\boldsymbol{K}_{T_4}$) are given by equatios (27) to (29):

$$\boldsymbol{K}_{T_1} = \begin{bmatrix} -0.0830 & 10.2109 & 0.4656 & 0.0060 \\ -0.1662 & -113.9739 & 3.6039 & 0.0271 \end{bmatrix}. \tag{27}$$

$$\boldsymbol{K}_{T_2} = \begin{bmatrix} -16.0304 & -56.4099 & 15.5965 & 0.6663 \\ 171.8166 & 600.5412 & -174.6060 & -7.1244 \end{bmatrix}. \tag{28}$$

$$\boldsymbol{K}_{T_4} = \begin{bmatrix} 0.3410 & -4.0121 & -0.2093 & -0.0030 \\ -0.2483 & 72.4547 & -1.6650 & -0.0101 \end{bmatrix}. \tag{29}$$

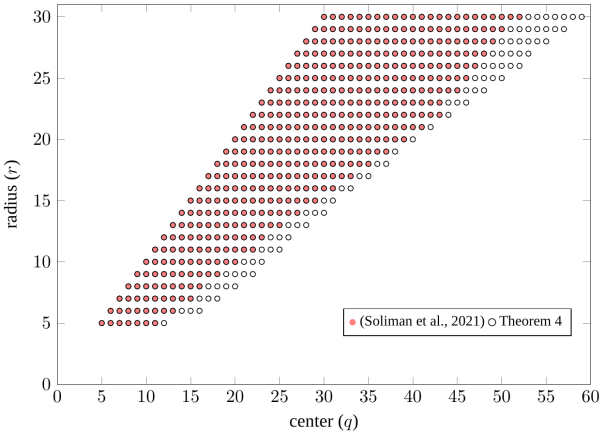
With respect to the controllers described in equations (27) to (29), there is an advantage in Theorem 4 in terms of the effort of the actuator to attend the same performance criteria, as Table 1 indicates. In this case, the controller $\boldsymbol{K}_{T_4}$ represents a more practical attainable proposal, in addition to allowing us to circumvent the inherent mathematical disadvantages of the characteristics of increasing exponentials that occur with increasing uncertainties in polytopic models, as in Theorem 1.

Figure 3 presents an analysis of the feasibility regions between Theorem 1 and Theorem 4, which shows in its axes the values of the disc parameters $D(q,r)$.

Table 1 - Comparative analysis between the norms of controllers.

Theorem	Norm
1	114.4855
2	651.4528
4	72.5850

Figure 3 - Feasibility regions corresponding to variations in parameters q and r of disc $D(q, r)$.



In the interval $r \in [5,\ 30]$, Figure 3, it was possible to map out the values of q in which the LMIs are feasible. Observe that Theorem 4 allows expansion of the project options due to the LMI structure, which includes slack variables ε that allows the relaxation the LMIs of Soliman et al. (2011). As a result, expanding regions of the complex plane become feasible, specially those regions far from the origin.

On the other hand, in order to illustrate the time performance of the designed controllers, we evaluate behavior of the angle deviation $\Delta\delta$ for the following cases:

Case 1: the actuators were 100 % active;
Case 2: the actuators presented joint failure with 40 % power in the PSSs.

In conclusion, it can be seen that a quicker time response to Theorem 2 - only feasible for the study Case 1. In contrast, there is not a dominance between the proposals of Theorem 1 and Theorem 4 (via uncertainty models), which alternate in transitory performance according to the polytope vertices.

The transitory behavior shown in Figure 4 is justified through the localization of the eigenvalues of the closed loop system for the analyzed theorems, considering only the positive semi-plan of the imaginary axis in the complex plan λ, (Figure 5).

Figure 4 - Time response of the state variable $\Delta\delta$: (a) vertex 1 and (b) vertex 4.

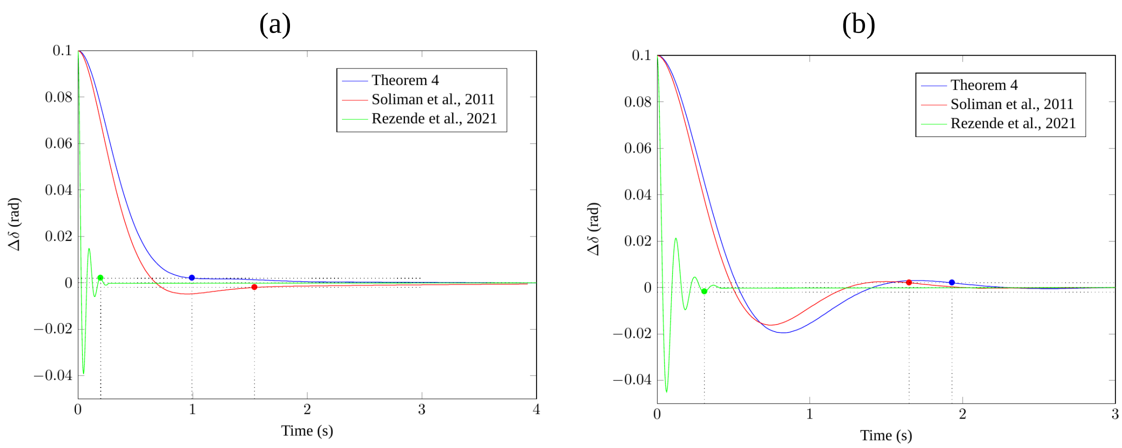
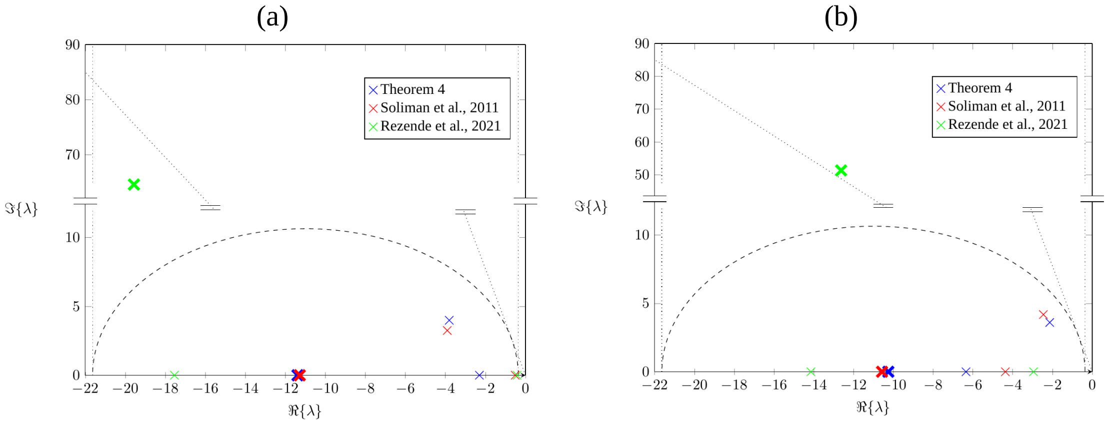


Figure 5 - Pole in closed loop with emphasis in the state variables $\Delta\delta$: (a) vertex 1 and (b) vertex 4.



In this case, it is possible to establish that Theorems 1 and 4 satisfy the performance region of disc $D(q, r)$; however in the control law project by Theorem 2, there was an attendance only for Vertex 1, because in the project phase it have been considered the matrices pair $(\boldsymbol{A},\ \boldsymbol{B})$, with the input matrix being a constant.

It is important to emphasize that in Figure 5 there is a dominance of three out of the four system's eigenvalues, which leads a characteristic equation in the form $D(\lambda) = (\lambda^2 + 2\xi'\omega_n'\lambda + \omega_n'^2)(\lambda + 1/\gamma)$. In this case, $|1/\gamma| \ngeq 5\xi'\omega_n'$ (vice-versa); however, if these are in the desired region for transient performance (like the stabilization time and dumping ratio), this will guarantee that the system shows the desired transitory performance, even if there are mixed responses for the eigenvalues.

Conclusions

This study analyzed the influence of uncertainty models on projects of the robust $\mathscr{D}$-LQR-LMI controllers kind. For this purpose, we proposed Theorem 4, which represents the LMI version of Theorem 3 considering the operation region of disc $D(q, r)$ from Theorem 1.

A numerical example of application in electrical systems of power illustrated the efficiency of our proposal in relation to others, which can be summarized by the following characteristics: (a) reduction in conservatism due to the increased regions of feasibility for variations of disc $D(q, r)$, (b) practical applicability, due to the reduced values of the controllers norms, and (c) computational cost due to the reduction of LMI lines to be programed for computing the state feedback gain.

Finally, the literature review points out as an area of investigation the use of fuzzy observers (based on LMIs) to estimate state variables of Takagi-Sugeno models.

Author contributions

R. da P. Caun participated in: Conceptualization, Project Management, Programs, Supervision, Validation, Visualization, and Writing (revision and editing). R. J de Carvalho participated in: Formal Analysis, Programs, Validation, Visualization, and Writing (preparation of the original draft). E. Assunção participated in: Conceptualization, Supervision, and Writing (revision and editing). W. R. B. M. Nunes participated in: Supervision, Validation, Visualization, and Writing (revision and editing). M. C. M. Teixeira participated in: Supervision and Writing (revision and editing). R. N. de Souza participated in: Validation, Visualization, and Writing (revision and editing).

Conflicts of interest

The authors declare no conflict of interest.

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