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Shear strength of steel fiber self-compacting concrete beams

Resistência ao cisalhamento de vigas de concreto autoadensável com adição de fibras de aço

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Abstract

The use of self-compacting concrete has increased for several reasons over recent decades but, mainly due to its high fluidity, which dispenses of the need for concrete vibrators, ease of casting, higher quality and better compacting, allowing the production of slender pieces, with a higher reinforcement ratio. However, even self-compacting concrete exhibits brittle failure behavior and low tensile and shear strength, issues that can be mitigated with the use of steel fibers. Aiming to investigate the shear strength in self-compacting concrete beams with steel fibers, this study presents a database collected from 113 experimental tests in the literature. Using the Root Mean Square Error (RMSE) and the Collins' Demerit Points Classification (DPC), five code-based equations and ten experimental based equations for the prediction of the shear capacity of SFRC beams were evaluated. The results show that, unlike concrete without the addition of fibers, increase in aggregate dimensions decreases the shear strength with the use of steel fibers in SCC beams. Additionally, the increase in fiber volume corresponds to an increase in concrete shear strength with a maximum compressive strength of 50 MPa. The results also demonstrate that the Root Mean Square Error (RMSE) is better for evaluating the precision but not the safety of the shear strength prediction equations, which are better determined by Collins' Demerit Points Classification (DPC). Code-based equations for ultimate shear strength prediction of fiber reinforced concrete beams presented results with satisfactory safety and economy.

Keywords: Self-compacting concrete. Steel fibers. Shear strength. Beams. Structural concrete.

Resumo

A utilização do concreto autoadensável tem crescido nas últimas décadas devido à diversos motivos, sendo os principais a elevada fluidez, que dispensa a utilização de vibradores mecânicos, a facilidade de concretagem e a maior qualidade do concreto, resultado do melhor adensamento, permitindo a concretagem em peças esbeltas e com maior taxa de armadura. Entretanto, mesmo sendo o concreto autoadensável, apresenta ruptura frágil, baixa resistência à tração e ao cisalhamento, fatores que podem ser amenizados com a utilização de fibras de aço. Para avaliar a resistência ao cisalhamento em vigas de concreto autoadensável com adição de fibras, este estudo apresenta dados coletados na literatura de 113 ensaios experimentais. Utilizando o método estatístico Raiz do Erro Quadrático Médio (RMSE) e a Classificação por Pontos de Demérito de Collins (DPC), foram avaliadas a aplicabilidade de cinco equações normativas e dez equações desenvolvidas em estudos experimentais para a estimativa da resistência ao cisalhamento de vigas de concreto convencional. Os resultados mostram que, diferente do concreto sem adição de fibras, o aumento do diâmetro do agregado diminui a resistência ao cisalhamento quando adicionadas fibras no CAA. Entretanto, o aumento do volume de fibras corresponde à um aumento da resistência ao cisalhamento em concretos com resistência à compressão inferiores a 50 MPa. Os resultados demonstram que o método RMSE é indicado para análise da precisão, mas não da segurança das equações de estimativa de resistência ao cisalhamento, que é melhor avaliada pela DPC. As equações de estimativa da resistência ao cisalhamento propostas em normas apresentaram resultados com satisfatória margem de segurança e economia, podendo ser empregadas no dimensionamento de estruturas.

Palavras-chave: Concreto autoadensável. Fibras de aço. Resistência ao cisalhamento. Vigas. Concreto estrutural.

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Introduction

Self-compacting concrete (SCC) is a type of concrete that can flow under its own weight and fill the formwork completely, maintaining homogeneity even in the presence of reinforcement and then consolidating without the need for vibration (GEIKER; JACOBSEN, 2019). The materials used for the production of self-compacting concrete are the same as for conventional vibrated concrete (CVC), but with a greater amount of fines (chemically active mineral additions or fillers), superplasticizer additives and/or viscosity modifiers, obtaining three properties: fluidity, necessary cohesion for the mixture to flow intact between steel bars (or passing ability) and resistance to segregation (EFNARC, 2002; TUTIKIAN; MOLIN, 2008).

Despite the great use of concrete in structures, SCC has its limitations, such as fragile behavior in its hardened state and low deformation capacity before rupture, due to the material's susceptibility to cracks and micro-cracks that occur under tensile stress. Such limitations can be solved by adding fibers to the concrete (FIGUEIREDO, 2011), which is not meant to increase the strength of the composite but serves as a stress transfer bridge, reducing the propagation of cracks and changing the behavior of the concrete in the post-cracking situation (BENTUR; MINDESS, 1990; RANDO JUNIOR; GUERRA; MORALES, 2019).

Studies found in the literature (PAUW; BUVERIE; MOERMAN, 2008; SUSETYO; GAUVREAU; VECCHIO, 2011) demonstrate that in beams with minimal transverse reinforcement, the use of steel fibers can equal, and even exceed, the shear strength provided by stirrups. Gali and Subramaniam (2018) incorporated steel fibers into self-compacting concrete, noting that even with the shear strength lower than conventional concrete, SCC with the addition of fibers reached levels of resistance comparable to CVC.

The equations currently proposed for estimating the shear strength of beams reinforced with steel fibers are mostly based on experimental results, some of which are adopted in design standards (AFGC, 2013; DEUTSCHER AUSSCHUSS FÜR STAHLBETON, 2013; CNR-DT, 2007; FIB, 2012; RILEM, 2003). Based on the above, the present work evaluates 15 equations proposed in the literature and in design standards for reinforced concrete beams with the addition of steel fibers, in terms of the shear strength of self-compacting concrete, by using a database elaborated from experimental tests presented in the literature.

Methods

Steel fibers are characterized by the fiber factor F, equation (1),

$$F = \frac{L_f}{D_f} V_f d_f, \tag{1}$$

where: L_f is the fiber's length (mm), D_f is the fiber's diameter (mm), V_f is the fiber's volume (%) and d_f is the bond fator.

According to Narayanan and Darwish (1987), the value D_f, which considers the connection between the concrete matrix and the fiber, can be considered to be 0.50 for smooth fibers, 0.75 for crimped fibers and 1.00 for hooked end fibers.

Ultimate shear strength estimates for fiber reinforced concrete beams

Table 1 presents equations for ultimate shear strength estimates for steel fiber reinforced concrete beams, extracted from scientific papers and concrete structure design codes. All of the parameters, presented in the equations (2)-(16), are described in the notation list.

Lantsoght (2019) noted that equations that separate the fibers' strength from the rest of the resistance mechanisms are not theoretically correct, as the fibers work together with other mechanisms. To develop a precise equation, it would be necessary to theoretically quantify the relationship between the fibers and resistance mechanisms and test them experimentally.

There are differences in shear strength capacity for conventional and self-compacting concretes with similar strengths due to the size and content of the coarse aggregates used (GALI; SUBRAMANIAM, 2018). Among the equations presented in Table 1, only equation (2) considers the aggregate dimensions as a shear resistance parameter (IMAM; VANDEWALLE; MORTELMANS, 1997).

The equations (12)-(13), proposed by the Association Française de Génie Civil (AFGC, 2013), the German guideline DAfStB (DEUTSCHER AUSSCHUSS FÜR STAHLBETON, 2013) and RILEM (2003), respectively, consider that the shear strength in beams without transverse reinforcement is the result of the action of two portions: one resisted by concrete and another resisted by steel fibers. Equation (15), presented by the fib Model Code (FIB, 2012), incorporates the effect of fibers, being the same adopted by the Italian guide CNR-DT (2007), equation (16), however, it considers a lower limit for V_{min}.

Table 1 – Equations for estimating the ultimate shear strength of concrete beams with fibers

References	Proposed equation	
Imam, Vandewalle and Mortelmans (1997)	$\begin{cases} V_u = 0.6bd\Psi\sqrt[3]{\omega}\left[f_c^{0.44} + 275\sqrt{\dfrac{\omega}{(a/d)^5}}\right] \\ \Psi = \dfrac{1+\sqrt{\dfrac{5.08}{d_a}}}{\sqrt{1+\dfrac{d}{(25d_a)}}} \\ \omega = \rho\,(1+4F) \end{cases}$	(2)
Narayanan and Darwish (1987)	$\begin{cases} V_u = bd\left[e\left(0.24f_{sp} + 80\rho\dfrac{d}{a}\right) + v_b\right] \\ v_b = 0.41\tau F \\ e = 2.8\dfrac{d}{a} \;\text{ if }\; \dfrac{a}{d} \le 2.8 \\ e = 1\dfrac{d}{a} \;\text{ if }\; \dfrac{a}{d} > 2.8 \end{cases}$	(3)
Ashour, Hasanain and Wafa (1992), based on the equation of ACI (1989)	$V_u = bd\left[\left(0.7\sqrt{f_c} + 7F\right)\dfrac{d}{a} + 17.2\rho\dfrac{d}{a}\right]$	(4)
Ashour, Hasanain and Wafa (1992), based on the equation of Zsutty (1971)	$\begin{cases} V_u = bd\left[\left(2.11\sqrt[3]{fc}\right)\left(\rho\dfrac{d}{a}\right)^{0.333}\right] \;\text{ if }\; \dfrac{a}{d} \le 2.5 \\ V_u = bd\left[\left(2.11\sqrt[3]{fc}\right)\left(\rho\frac{d}{a}\right)^{0.333}\dfrac{2.5a}{d} + v_b\left(2.5 - \dfrac{a}{d}\right)\right] \;\text{ if }\; \dfrac{a}{d} > 2.5 \end{cases}$	(5)
Kwak *et al*. (2002)	$\begin{cases} V_u = bd\left[3.7e\,(f_{sp})^{\frac{2}{3}}\left(\rho\dfrac{d}{a}\right)^{\frac{1}{3}} + 0.8\,v_b\right] \\ e = 3.4\dfrac{d}{a} \;\text{ if }\; \dfrac{a}{d} \le 3.4 \\ e = 1 \;\text{ if }\; \dfrac{a}{d} > 3.4 \end{cases}$	(6)
Khuntia, Stojadinovic and Goel (1999)	$V_u = bd\,(0.167 + 0.25F)\sqrt{f_c}$	(7)
Sharma (1986)	$\begin{cases} V_u = bd\left(k' f_{sp}\left(\dfrac{d}{a}\right)^{0.25}\right) \\ k' = 1 \;\text{ or }\; k' = \dfrac{2}{3} \;\text{ for direct and indirect tensile strength test, respectively} \\ k' = \dfrac{9}{4} \;\text{ if }\; f_{sp} \;\text{ is determined by rupture modulus} \\ \text{or using equation }\; f_{sp} = 0.79f_c^{0.5} \end{cases}$	(8)
Sarveghadi *et al*. (2019)	$V_u = \left[\rho + \dfrac{\rho}{v_b} + \dfrac{1}{a/d}\left(\dfrac{\rho f_{sp}\;(\rho+2)\left(f_{sp}\dfrac{a}{d} - \dfrac{3}{v_b}\right)}{a/d}f_{sp}\right) + v_b\right]bd$	(9)

Table 1 – Equations for estimating the ultimate shear strength of concrete beams with fibers (cont.)

References	Proposed equation	
Greenough and Nehdi (2008)	$V_u = \left[0.35\left(1+\sqrt{\frac{400}{d}}\right) f_c^{0.18}\left((1+F)\rho\frac{d}{a}\right)^{0.4} + 0.9v_b\right] bd$	(10)
Swamy, Jones and Chiam (1993), from ACI (2014)	$\begin{cases} V_u = 0.9v_b + V_{Rd,c} \\ V_{Rd,c} = 0.167\sqrt{f_c}bd \end{cases}$	(11)
Association Française de Génie Civil - AFGC (2013)	$\begin{cases} V_u = V_{Rd,c} + V_{Rd,f} \\ V_{Rd,c} = \frac{0.21}{\gamma_{cf}\gamma_E} f_c^{1/2} bd \\ V_{Rd,f} = \frac{bz\sigma_{Rd,f}}{tan\theta} \end{cases}$	(12)
Deutscher Ausschuss Für Stahlbeton - DAFSTB (2013)	$\begin{cases} V_u = V_{Rd,c} + V_{Rd,cf} \\ V_{Rd,c} = \frac{C_{Rd,c}}{\gamma_c} k(100\rho f_{ck})^{1/3} bd > V_{Rd,c,\,min} \\ V_{Rd,cf} = \frac{\propto_c^f f_{ctR,u}^f bd}{\gamma_{ct}^f} \\ f_{ctR,u}^f = k_F^f k_G^f 0.37 f_{cfIk,L2}^f \\ k_G^f = 1 + 0.5A_{ct}^f \leq 1.7 \\ A_{ct}^f = b \times min(d, 1.5m) \\ k = 1 + \sqrt{\frac{200}{d}} \end{cases}$	(13)
Rilem (2003)	$\begin{cases} V_u = V_{Rd,c} + V_{Rd,cf} \\ V_{Rd,c} = 0.12k(100\rho f_{ck})^{1/3} bd \\ V_{Rd,cf} = 0.7k_f k \tau_{fd} bd \\ k_f = 1 + n\left(\frac{h_f}{b}\right)\left(\frac{h_f}{d}\right) \leq 1.5 \\ n = \frac{b_f - b}{h_f} \leq 3 \text{ and } n \leq \frac{3b}{h_f} \\ \tau_{fd} = 0.12 f_{Rk,4} \end{cases}$	(14)
Model Code - FIB (2012)	$\begin{cases} V_u = V_{Rd,f} = \frac{C_{Rd,c}}{\gamma_c} k\left(100\rho_l\left(1+7.5\frac{f_{Ftuk}}{f_{ctk}}\right) f_{ck}\right)^{\frac{1}{3}} bd \\ f_{ctk} = 0.3(f_{ck})^{2/3} \text{ if } f_c \leq 50\,MPa \\ f_{ctk} = 2.12\ln(1+0.1(f_{ck}+8\,MPa)) \text{ if } f_c > 50\,MPa \end{cases}$	(15)
CNR-DT (2007)	$\begin{cases} V_u = V_{Rd,f} \geq V_{min} \\ V_{min} = 0.035k^{\frac{2}{3}} f_{ck}^{1/3} bd \end{cases}$	(16)

Source: The authors.

With the exception of the Association Frnçaise de Génie Civil (AFGC, 2013), the code-based equations are valid for conventional concrete beams and longitudinal reinforcement ratio $\rho \leq 2\%$. Among these equations, the proposals by the Association Française de Génie Civil (AFGC, 2013), Deutscher Ausschuss für Stahlbeton (DAFSTB, 2013) and Rilem (2003) consider the shear strength portion related to concrete regardless of the fiber resisted portion.

Development of database

The database developed for this study contains the results of 113 experimental tests of self-compacting concrete beams with the addition of steel fibers, obtained in the literature (GREENOUGH; NEHDI, 2008; DING; YOU; JALALIC, 2011; HELINCKS *et al*., 2011; DING *et al*., 2012; ALTAAN; AL-NEIMEE, 2012; FRITIH *et al*., 2013; EL-DIEB; EL-MAADDAWY; AL-RAWASHDAH,, 2014; AOUDE; COHEN, 2014; CUENCA; ECHEGARAY-OVIEDO; SERNA, 2015; NING *et al*., 2015; RAWASHDEH, 2015; ADAM; SAID; EKKARIB, 2016; HAMEED; AL-SHERRAWI, 2018; KANNAM *et al*., 2018; PRAVEEN; RAO, 2019).

The beams that make up the database only had longitudinal reinforcement but no stirrups and were tested on three or four point bending tests. The complete database description can be found in Appendix.

For comparison purposes, the volume of fibers used in the concrete was transformed into a percentage of the fiber volume (V_f), considering the density of steel fibers equal to 7.850 kg/m^3

In the analysis of the code-based equations (12)-(16), some parameters were adopted according to the recommendations in EUROCODE 2 (EUROPEAN UNION, 2004): $C_{Rd,c}$ equal to 0.15, and the safety coefficients that consider long-term effects $\gamma_c = 1.5$, $\gamma_{ct}^f = 1.25$, $\propto_c^f = 0.85$, and $k_F^f = 0.5$ for shear. The fib Model Code (FIB, 2012) recommends $C_{Rd,c} = 0.18$, which is the value that is also used in the Italian code.

Furthermore, EUROCODE 2 (EUROPEAN UNION, 2004) recommends that loads applied at a distance of 0.5d $\leq a_v \leq 2d$ must be reduced by a coefficient $\beta = \dfrac{a_v}{2d}$, where a_v is the shear span. For beams that do not fit this condition, the value of β equal to 1.0 was adopted. Thus, as Lantsoght (2019) recommended, for all codes except the French guideline, the β coefficient was considered when calculating shear strength.

For equation (12), from AFGC (2013), the angle of the compression strut θ was considered to be equal to 30°. The safety factors of the material $(\gamma_{cf}\gamma_E)$ were disregarded since the equations were not being used for design but working with experimental data. According to Lantsoght (2019), the value of K can be approximated as 1.25. The variables $f_{cfIk,L2}^f$ and $f_{Rk,4}$ were considered equal to f_{sp}, since it is a parameter that can only be obtained through experimental tests; as it depends on data such as concrete deformation, the opening of cracks and tension after cracking of the beams.

When the value of the tensile strength of the concrete f_{sp} was not presented in the article, it was calculated as recommended by Sharma (1986), using equation (17)

$$f_{sp} = 0.79 f_c{}^{0.5}. \tag{17}$$

Fritih *et al*. (2013) used smooth and amorphous steel fibers with a thickness of 1.6 mm. As the authors do not presented in the article the diameter of fibers used, for calculating the fiber F factor a diameter of 0.8 mm was considered for calculating shear strength in this work. In Greenough and Nehdi's (2008) research, some of the beams analyzed contained smooth fibers with flattened ends. The d_f value was not found for this type of fiber, so $d_f = 0.5$ was used, which considers the fiber to be entirely smooth. AlTaan and Al-Neimee (2012) use 16 mm shell-shaped fibers and provided the equivalent diameter of 0.78 mm. For this type of fiber, $d_f = 0.5$ was also used.

Hameed and Al-Sherrawi (2018) did not present specific data on the compressive strength of concrete f_c, declaring that after three mixtures and corrections in the proportions, the concrete reached the required strength of 40 MPa, verified through testing concrete cubes; this is equivalent to 34 MPa for cylindrical specimens.

Data analysis methods

The experimental results of the database were associated with several characteristics of the beams. From these data, trends in beam behavior were verified with the use of fibers in self-compacting concrete and the capacity of the code-based equations to estimate the shear strength of these beams.

As there is a significant variation in beams geometry in this database, it was decided to carry out the analysis according to the ultimate shear stresses. The shear stress is given by the ultimate shear force divided by the area of the cross-section, described in the equation (18),

$$\tau_u = \frac{V_u}{bd}. \tag{18}$$

Also, due to the disparity between the compressive strength of concrete used by the studies considered, the shear stress normalized by the compressive strength's square root was used (τ_n), given by

$$\tau_n = \frac{\tau_u}{\sqrt{f_c}}. \tag{19}$$

In order to evaluate the applicability of the self-compacting concrete equations in the literature (SHARMA, 1986; NARAYANAN; DARWISH, 1987; ASHOUR, HASANAIN; WAFA,1992; SWAMY; JONES; CHIAM, 1993; IMAM; VANDEWALLE; MORTELMANS, 1997; KHUNTIA; STOJADINOVIC; GOEL, 1999; KWAK *et al.*, 2002; GREENOUGH; NEHDI, 2008; SARVEGHADI *et al.*, 2019) and design codes (AFGC, 2012; CNR-DT, 2007; DEUTSCHER AUSSCHUSS FÜR STAHLBETON, 2013; FIB, 2012; RILEM, 2003), two methods were used: the Root of the Mean Square Error (RMSE) and the Demerit Points Classification (DPC).

The RMSE measures the difference between a sample's values or population predicted by a model and the observed values, called residue. According to Evans (2013), because RMSE is expressed in the same units as the data being compared, it allows more objective comparisons. The RMSE equation is given by

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{n_o}(A_t - F_t)^2}{n_o}}, \tag{20}$$

where: A_t is the observed value (measured), F_t is the calculated value, and n_o is the number of observations (data).

In equation (20), the difference between measured and calculated values $(A_t - F_t)$ is squared, so the RMSE assigns a greater weight to larger errors, being ideal for measuring the precision of the estimate made by equations where large errors are particularly undesirable, as they can mean under- or over-dimensioning of the beams.

The DPC method presented by Collins (2001), considers the safety and economy of the equations through the relationship between the shear force observed in experimental tests (V_u) and those estimated by the codes (V_r), establishing a penalty for each beam analyzed. The greatest penalty is applied to the least secure beams, in which the experimental strength has not reached half of the estimated theoretical value. As the V_u/V_r ratio increases, the equation is considered to be safer, and the penalty is reduced. However, beams classified as conservative also receive a penalty value as they are oversized beams and considered uneconomical.

The final penalty value is obtained by the sum of the number of beams that resulted in each interval multiplied by the respective penalty value. Thus, the DPC method considers that the higher the penalty, the worse is the use of that equation for dimensioning the beams. Table A shows the ranges of the DPC classification.

Table 2 – DPC classification

V_u/V_r	**Classification**	**Penalty**
$V_u/V_r \leq 0.5$	extremely dangerous	10
$0.5 < V_u/V_r \leq 0.65$	dangerous	5
$0.65 < V_u/V_r \leq 0.85$	low safety	2
$0.85 < V_u/V_r \leq 1.3$	appropriate safety	0
$1.3 < V_u/V_r \leq 2$	conservative	1
$V_u/V_r > 2$	extremely conservative	2

Source: Collins (2001).

Results and discussion

Database parameters

Table 3 shows the minimum and maximum values of several characteristics of the analyzed beams.

Table 3 – Parameter's range in database

Parameter	**Minimum**	**Maximum**
b (mm)	90	200
h (mm)	150	450
d (mm)	122	425
a (mm)	250	1400
l total (mm)	1000	3000
ρ_l (%)	0.35	5.88
a/d	0.60	5.53
V_f (%)	0.25	1.57
fc (MPa)	27	100

Source: The authors.

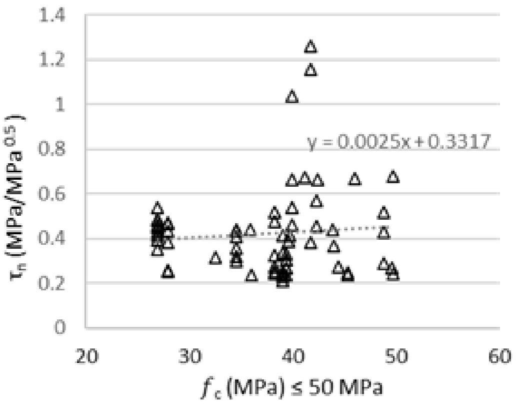
In the studies evaluated, hooked end, flattened end, smooth, beaded, and shell-shaped end fibers were used. The most used fiber type was hooked end, comprising around 70% of the analyzed experiments.

Most papers used concrete with compressive strength below 50 MPa and longitudinal reinforcement rate ρ as 2%, however, there were a large number of beams with a reinforcement ratio close to 6%, justified by the need to avoid flexural failure.

Most of the beams had a depth d of 200 mm and the frequent a/d ratio used was 3.5. The maximum fiber volume used in concretes was 1.6%, limited by the workability of self-compacting concrete.

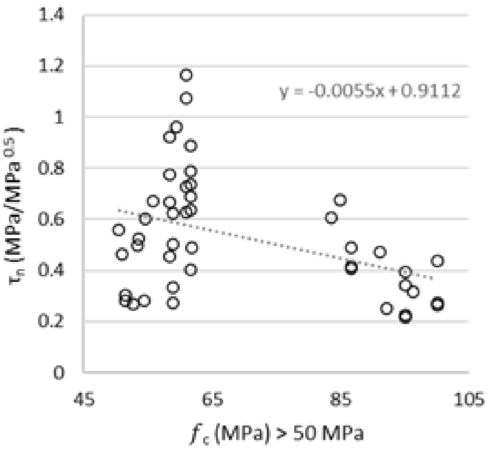
Figure 1 shows the relationship between the concrete's compressive strength and the normalized shear stress resisted by the beams, for concretes with axial compressive strength up to 50 MPa. It was observed that there was no increase in shear strength for greater compressive strengths, probably due to the rupture mechanism relying on the fibers.

Figure 1 – Relationship between τ_n and f_c for concretes with $f_c \leq 50$ MPa.



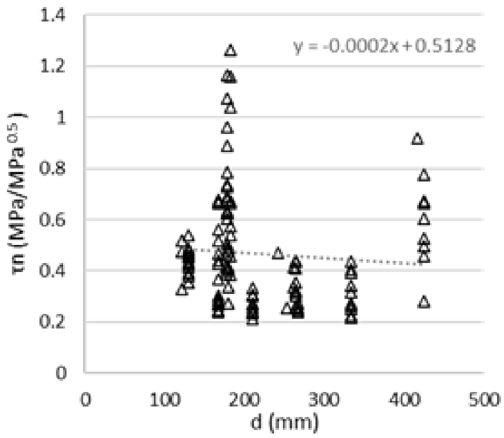
Source: The authors.

Figure 2 – Relationship between τ_n and f_c for concretes with $f_c > 50$ MPa.



Source: The authors.

Figure 3 – Relationship between τ_n and effective depth.



Source: The authors.

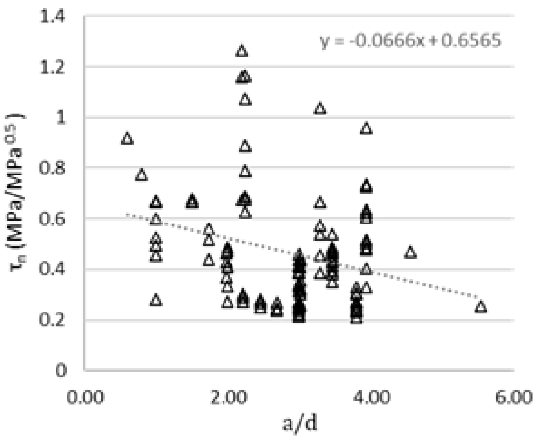
For concretes with f_c greater than 50 MPa, Figure 2, here is a reduction in shear strength with an increase in a concrete's compressive strength. This behavior can be explained by the way high-performance concretes crack: their rupture is more fragile than conventional strength concretes, generating smoother cracks, which reduce the shear transfer since there is less aggregate interlocking (SHAH; AHMAD, 2007; PERERA; MUTSUYOSHI, 2013).

In Figure 3, it can be seen that the increase in the depth of the beam decreases the ultimate shear stress, demonstrating the occurrence of the scale effect. Shoaib, Lubell and Bindiganavile (2014) reported the same behavior for conventional concrete beams with the addition of steel fibers, as well as Lantsoght (2019). However the authors emphasizes the need for further studies with full-scale beams to assess the relationship between shear strength and scale effectss in beams with the addition of steel fibers.

Regarding the shear span to an effective depth ratio a/d, Figure 4 shows that its increase results in a decrease of the shear strength, similar to the results obtained for conventional concrete beams and steel fibers by Lantsoght (2019) and for concrete beams self-compacting with the addition of steel fibers from AlTaan and Al-Neimee (2012) and Adam, Said and Elkarib (2016). This effect is explained by the reduction in the contribution of the arch action effect in the beams' strength when there is an increase in a/d.

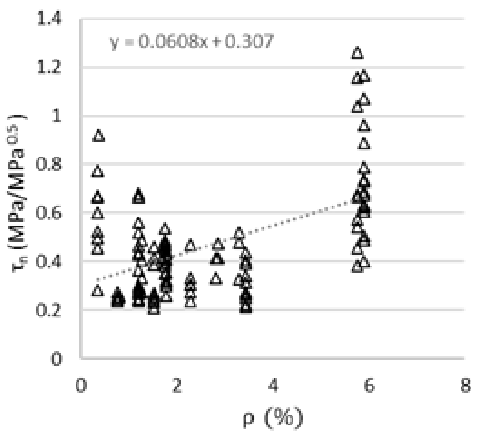
The dowel effect on self-compacting concrete beams with steel fibers can increase the longitudinal reinforcement rate, Figure 5. Li *et al.* (2019), in their study of lightweight concrete beams with the addition of steel fibers, suggested that the increase in shear strength in these cases is due to the combination of the increased roughness in concrete cracks, due to the addition of fibers, and the dowel effect.

Figure 4 – Relationship between τ_n and a/d.



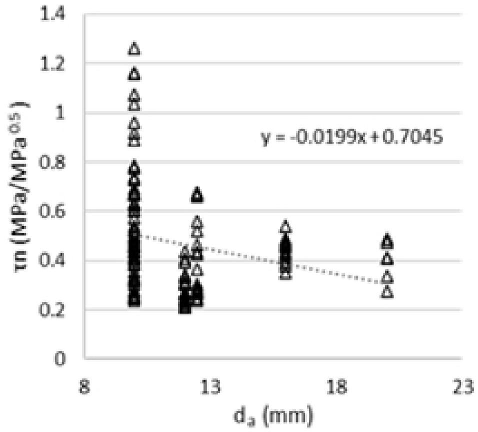
Source: The authors.

Figure 5 – Relationship between τ_n and ρ_l.



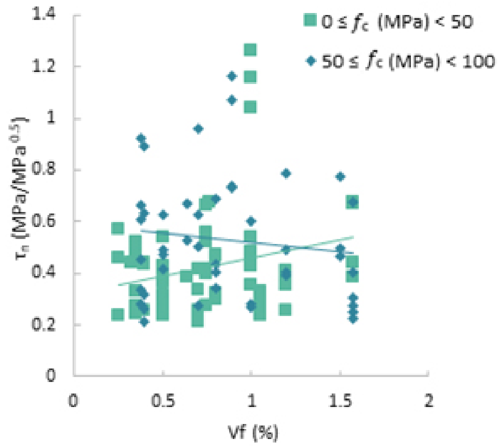
Source: The authors.

Figure 6 – Relationship between τ_n and d_a.



Source: The authors.

Figure 7 – Influence of fiber content and fc on shear strength.



Source: The authors.

The aggregate interlock showed an inverse relationship to what occurs in concrete without the addition of fibers. Figure 6 shows that the use of coarse aggregate of larger granulometry resulted in a reduction in shear strength. Lantsoght (2019) attributed this behavior to the greater uniformity in the concrete matrix with aggregates of smaller dimension, which results in a better connection between the fibers and concrete.

Figure 7 shows the relationship between the ultimate normalized shear stress and the volume of fibers used, divided into two groups, according to the compressive strength of the concrete f_c. It can be observed that an increase in fiber volume results in higher shear strength when $fc < 50$ MPa, corresponding to group I of the NBR 8953 standard (ABNT, 2015).

Shear strength decreases when the fiber content increases for concretes with compressive strength higher than 50, diverging from studies with conventional concrete beams (PANSUK, 2017; SUJIVORAKUL, 2012) and self-compacting concrete (RAWASHDEH, 2015). However, according to Smarzewski (2018), shear strength not only depends on the volume of fibers but also on their geometry and type of anchorage. Wille, Kim and Naaman (2010) evaluated ultra-high performance concrete specimens with the addition of various types of fibers (FRUHPC) and showed that the increase in the volume of hooked-end fibers resulted in a decrease in the resistant capacity, since the anchoring created stress peaks in the concrete matrix that generated micro-cracks.

Another possibility, presented by Larsen and Thorstensen (2020), who tested FRUHPC specimens under flexure, is that a high volume of fibers can increase the concentration of fibers in specific points in the concrete mix, creating air bubbles that reduce the resistance of the matrix of the concrete.

Analysis of the empirical equations

With the empirical equations (2)-(11) presented in Table 1, theoretical shear capacities were calculated and compared to experimental results using the RMSE and DPC methodologies; the results are shown in Figure 8.

The equations with the most significant inaccuracy were those defined by Ashour, Hasanain and Wafa (1992), based on ACI318-89, equation (5), and Kwak *et al*. (2002), equation (6), that presented the worst results for RMSE and DPC. The equation presented by Ashour, Hasanain and Wafa (1992), based on ACI318-89, equation (6), did not take into account the dowel effect and overestimated the resistant capacity of short beams. The equation presented by Kwak *et al*. (2002), equation (6), overestimated the resistance of beams with a low longitudinal reinforcement rate and overestimated the strength in beams with higher rates.

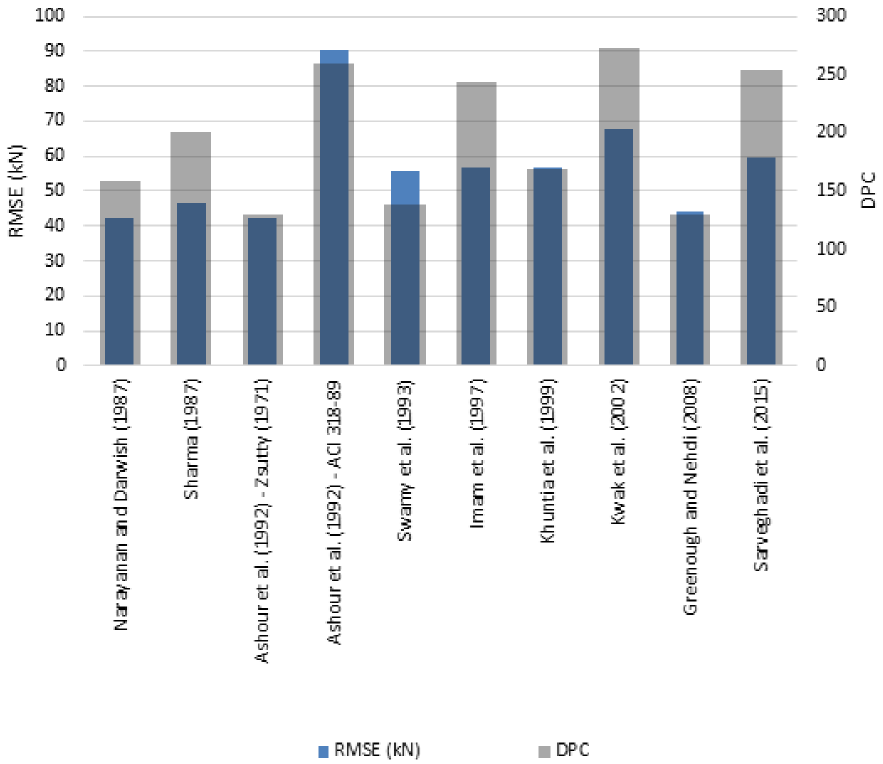
It was possible to observe disparities between the DPC method's penalties and the error by the RMSE method, as in the equations of Imam, Vandewalle and Mortelmans (1997) and Sarveghadi *et al*. (2019), equations (2) and (9), respectively. These are the results from the analysis method: for the DPC, the equations with the highest amounts of beams in the dangerous ranges have the most significant penalty, whereas the RSME considers that the difference between the resistance value obtained experimentally and the one calculated is weighted according to the magnitude of this difference, regardless of whether this value is positive (more conservative) or negative (less secure).

An equation with a minor error was presented by Ashour, Hasanain and Wafa (1992), based on the equation of Zsutty (1971), equation (5), which also has the lowest penalty for the DPC scale. This penalty, however, is equal to the equation proposed by Greenough and Nehdi (2008), equation (10). Narayanan and Darwish (1987), equation (3), also has a low RMSE value, but the DPC scale penalty is slightly higher than Greenough and Nehdi (2008).

Analysis of the code-based equations

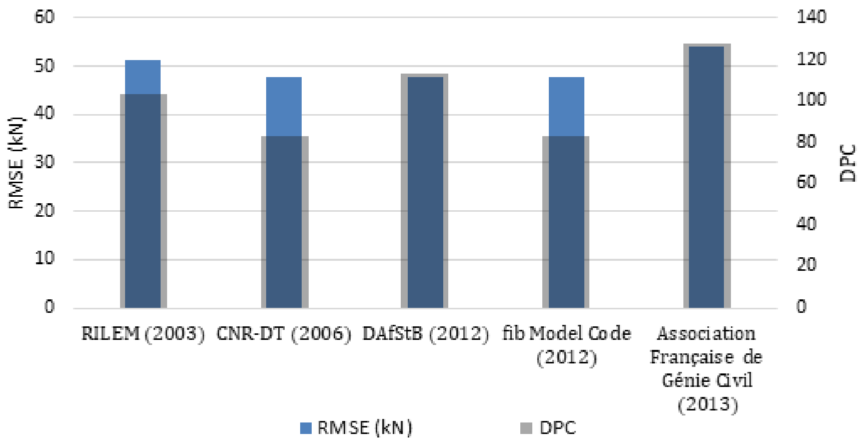
Figure 9 presents the results for the errors obtained by RMSE and the penalties on the DPC scale for the code-based equations presented in Table 1. The equations proposed by the fib Model Code (FIB, 2012) and CNR-DT (2007), equations (15) and (16), respectively, stand out for presenting the minor errors and penalties on the DPC scale among the code provisions. It is noteworthy that the Italian code CNR-DT (2007) presented results equal to those of the fib Model Code (FIB, 2012) because none of the beams analyzed in the database had a calculated resistance lower than the Vmin limit defined in this code.

Figure 8 – RSME and DPC results for empirical equations.



Source: The authors.

Figure 9 – RMSE and DPC for the code-based equations.



Source: The authors.

In addition to not having any beams in the dangerous and extremely dangerous ranges of the DPC scale, the fib Model Code (FIB, 2012) and the CNR-DT (2007) also resulted in the least number of equations in the extremely conservative range thus resulting in a lower penalty. However, its results still vary enough to reach an error value using the RMSE method close to the other code provisions.

The RMSE values for the code-based equations were above the empirical equations that showed good results, equations (3), (5) and (10), Narayanan and Darwish (1987), Ashour, Hasanain and Wafa (1992), based on the equation of Zsutty (1971) and Greenough and Nehdi (2008), respectively. This means that they estimate worse results than empirical equations.

None of the code-based equations presented results in the extremely dangerous range and only a few results in the extremely conservative range, except for equation (12), which was proposed by the AFGC (2013), which has the highest error and penalty value on the DPC scale. This equation attributes a smaller portion of shear strength to the fibers' action compared to other code provisions.

Even so, for the design of self-compacting concrete beams with steel fibers, the code-based equations are safer and more economical than the empirical equations since they have lower scores on the DPC scale. However, concerning the precision of the estimated shear strength, empirical equations were given by Narayanan and Darwish (1987), Ashour, Hasanain and Wafa (1992), based on the equation of Zsutty (1971) and Greenough and Nehdi (2008), equations (3), (5) and (10), respectively.

Conclusions

In this research, the results of a database containing 113 self-compacting concrete beams with the addition of steel fibers were analyzed and compared with equations designed to calculate the shear strength in conventional concrete beams with fibers. The main conclusions are:

- The increase in f_c only corresponds to an increase in shear strength in concretes with a strength of up to 50 MPa, since high-performance concretes' behavior in terms of shear strength is different from normal strength concrete, resulting in their rupture being more fragile.
- Increasing the aggregates size decreases the shear strength in self-compacting concrete beams with the addition of steel fibers, as they reduce the uniformity in the concrete matrix, making the connection between the fibers and the concrete difficult. However, there have been a few studies with CAA considering the addition of aggregates larger than 12.5 mm; this requires further investigation of the relationship between the volume of the fibers and the aggregate size.
- The RMSE method does not measure the safety of the analyzed equations. It considers that the difference between the resistance value obtained experimentally and that calculated has a weight according to the magnitude of this difference, regardless of whether this value is positive (more conservative) or negative (less safe). Thus, it is concluded that this method is more indicative for evaluating the precision of the equations. For safety assessment, the DPC method should be used.
- From the empirical equations that have been evaluated and analyzed, the safest and most accurate equation considering the DPC scale and the error by the RMSE method was proposed by Ashour, Hasanain and Wafa

(1992), based on the Zsutty equation (1971). Other equations with similar results were proposed by Narayanan and Darwish (1987) and Greenough and Nehdi (2008), the first one being more precise and the second more secure.

- The empirical equations with the worst performances were the ones presented by Ashour, Hasanain e Wafa (1992), based on the ACI (1989) equation, Imam, Vandewalle and Mortelmans (1997), Kwak *et al*. (2002), and Sarveghadi *et al*. (2019). These are not recommended for use in self-compacting concrete with the addition of fibers.
- Through analysis by the DPC method, in general, all of the code-based equations presented safer results than the empirical equations, which were indicated for the design of beams. As for the RMSE method, it is safe to say that the equations presented by Ashour, Hasanain e Wafa (1992), based on the equation of Zsutty (1971), Narayanan and Darwish (1987), and Greenough and Nehdi (2008), are better for estimating shear strength. Although the results obtained are satisfactory, it is worth mentioning that due to the random distribution of fibers in the concrete, the need for transverse reinforcement must be evaluated by the designer.
- The AFGC (2013) equations results were very conservative when compared to other code provisions. The code-based equations with the best results, both on the DPC scale and by the RMSE method, were presented by the fib Model Code (FIB, 2012) and the CNR-DT (2007).

Acknowledgments

The authors would like to acknowledgment the support of the Materials and Structures Research Group (GPMAES) and the Federal University of Technology - Paraná for this research.

Appendix

The complete database description can be found in Table A.

Table A – Database of experimental results from literature of SFRC beams with longitudinal reinforcement without stirrups failing in shear

Beam	b (mm)	h (mm)	d (mm)	ρ_1	a (mm)	d_a (mm)	f_c (MPa)	V_f (%)	L_f (mm)	D_f (mm)	V_u (kN)
Helincks *et al*. (2011)											
SF30-1	100	150	130	0.0174	450	16	58.4	0.38	30	0.55	41.12
SF30-2	100	150	130	0.0174	450	16	58.4	0.38	30	0.55	39.25
SF30-3	100	150	130	0.0174	450	16	58.4	0.38	30	0.55	45.06
SF30-4	100	150	130	0.0174	450	16	58.4	0.38	30	0.55	35.05
SF55-1	100	150	130	0.0174	450	16	58.9	0.70	30	0.55	47.92
SF55-2	100	150	130	0.0174	450	16	58.9	0.70	30	0.55	53.79
SF55-3	100	150	130	0.0174	450	16	58.9	0.70	30	0.55	46.26
SF55-4	100	150	130	0.0174	450	16	58.9	0.70	30	0.55	42.69
SF70-1	100	150	130	0.0174	450	16	61	0.89	30	0.55	49.40
SF70-2	100	150	130	0.0174	450	16	61	0.89	30	0.55	44.28
SF70-3	100	150	130	0.0174	450	16	61	0.89	30	0.55	47.60
SF70-4	100	150	130	0.0174	450	16	61	0.89	30	0.55	38.81
Fritih *et al*. (2013)											
A-FRSCC	150	280	253	0.0081	1400	10	42.3	0.25	30	0.80	63.10
B-FRSCC	150	280	242	0.0228	1100	10	42.3	0.25	30	0.80	111.30
Greenough and Nehdi (2008)											
S-HE-50-0.5	200	300	265	0.0178	800	10	43.9	0.50	50	1.00	90.85
S-HE-50-0.75	200	300	265	0.0178	800	10	39.7	0.75	50	1.00	105.60
S-HE-50-1.0	200	300	265	0.0178	800	10	41.7	1.00	50	1.00	148.90
S-FE-50-0.5	200	300	265	0.0178	800	10	45.3	0.50	50	1.00	115.45
S-FE-50-0.75	200	300	265	0.0178	800	10	44	0.75	50	1.00	144.10

Table A – Database of experimental results from literature of SFRC beams with longitudinal reinforcement without stirrups failing in shear (cont.)

Beam	b (mm)	h (mm)	d (mm)	ρ_1	a (mm)	d_a (mm)	f_c (MPa)	V_f (%)	L_f (mm)	D_f (mm)	V_u (kN)
Greenough and Nehdi (2008)											
S-FE-50-1.0	200	300	265	0.0178	800	10	39.9	1.00	50	1.00	146.75
S-FE-30-0.5	200	300	265	0.0178	800	10	44.4	0.50	30	0.70	106.70
S-FE-30-0.75	200	300	265	0.0178	800	10	42.4	0.75	30	0.70	122.90
S-FE-30-1.0	200	300	265	0.0178	800	10	41.7	1.00	30	0.70	151.35
Ning *et al*. (2015)											
BS-A-SF30	200	300	266.67	0.0076	800	10	53.6	0.38	60	0.75	93.00
BS-A-SF50	200	300	266.67	0.0076	800	10	53.4	0.64	60	0.75	99.00
BS-B-SF30	200	300	266.67	0.0076	800	10	61.9	0.38	60	0.75	103.00
BS-B-SF50	200	300	266.67	0.0076	800	10	54.7	0.64	60	0.75	107.50
Ding *et al*. (2012)											
SF20 -∞	100	150	122	0.0330	480	10	36	0.25	60	0.75	24.00
SF40 -∞	100	150	122	0.0330	480	10	32.5	0.50	60	0.75	36.11
SF60 -∞	100	150	122	0.0330	480	10	41.2	0.76	60	0.75	37.33
Ding, You and Jalalic (2011)											
SFSCCB25 -∞ a	200	300	262	0.0281	786	10	35.9	0.32	35	0.55	105.26
SFSCCB25 -∞ b	200	300	262	0.0281	786	10	41.7	0.64	35	0.55	140.56
Aoude and Cohen (2014)											
M15-0.5%	125	250	210.52	0.0153	800	12	59.4	0.50	30	0.55	43.00
M15-1.0%	125	250	210.52	0.0153	800	12	51.5	1.00	30	0.55	48.00
M15-1.5%	125	250	210.52	0.0153	800	12	55.8	1.50	30	0.55	46.00
M15-0.5%H	125	250	210.52	0.0153	800	12	49.6	0.50	30	0.38	45.00
M15-0.75%H	125	250	210.52	0.0153	800	12	46	0.75	30	0.38	48.00
M15-1.5%5D	125	250	210.52	0.0227	800	12	52.8	1.50	60	0.92	52.00
M20-0.75%	125	250	210.52	0.0227	800	12	49.7	0.75	30	0.55	44.00
M20-1.0%	125	250	210.52	0.0227	800	12	51.5	1.00	30	0.55	58.00
M20-1.0%A	125	250	210.52	0.0227	800	12	54.5	1.00	30	0.55	59.00
M20-1.5%A	125	250	210.52	0.0227	800	12	50.5	1.50	30	0.55	62.00
Hameed and Al-Sherrawi (2018)											
B02-SF0.5-SH	150	200	177	0.0151	530	10	40	0.50	50	1.05	65.00
B03-SF0.75-SH	150	200	177	0.0151	530	10	40	0.75	50	1.05	70.00
B04-SF1-SH	150	200	177	0.0151	530	10	40	1.00	50	1.05	77.50
El-Dieb, El-Maaddawy and Al-Rawashdah (2014)											
S28-VF1	120	220	182	0.0575	600	10	28	0.40	35	0.55	62.50
S28-VF2	120	220	182	0.0575	600	10	28	0.80	35	0.55	77.00
S28-VF3	120	220	182	0.0575	600	10	28	1.20	35	0.55	120.00
D28-VF1	120	220	182	0.0575	400	10	28	0.40	35	0.55	78.00
D28-VF2	120	220	182	0.0575	400	10	28	0.80	35	0.55	146.00
D28-VF3	120	220	182	0.0575	400	10	28	1.20	35	0.55	134.00
S100-VF1	120	220	182	0.0575	600	10	100	0.40	35	0.55	84.00
S100-VF2	120	220	182	0.0575	600	10	100	0.80	35	0.55	100.00
S100-VF3	120	220	182	0.0575	600	10	100	1.20	35	0.55	125.00

Table A – Database of experimental results from literature of SFRC beams with longitudinal reinforcement without stirrups failing in shear (cont.)

Beam	b (mm)	h (mm)	d (mm)	ρ_1	a (mm)	d_a (mm)	f_c (MPa)	V_f (%)	L_f (mm)	D_f (mm)	V_u (kN)
AlTaan and Al-Neimee (2012)											
M0.35-1.5-1	200	150	167	0.0120	250	12.5	38.3	0.35	16	0.78	137.30
M0.35-1.5-2	200	150	167	0.0120	290	12.5	38.3	0.35	16	0.78	90.70
M0.35-1.5-3	200	150	167	0.0120	330	12.5	38.3	0.35	16	0.78	75.90
M0.35-1.5-4	200	150	167	0.0120	370	12.5	38.3	0.35	16	0.78	56.40
M0.35-1.5-5	200	150	167	0.0120	410	12.5	38.3	0.35	16	0.78	51.40
M0.35-1.5-6	200	150	167	0.0120	450	12.5	38.3	0.35	16	0.78	49.10
M0.7-1.7-1	200	150	167	0.0120	250	12.5	39.1	0.70	16	0.78	139.30
M0.7-1.7-2	200	150	167	0.0120	290	12.5	39.1	0.70	16	0.78	108.50
M0.7-1.7-3	200	150	167	0.0120	330	12.5	39.1	0.70	16	0.78	89.60
M0.7-1.7-4	200	150	167	0.0120	370	12.5	39.1	0.70	16	0.78	60.60
M0.7-1.7-5	200	150	167	0.0120	410	12.5	39.1	0.70	16	0.78	55.90
M0.7-1.7-6	200	150	167	0.0120	450	12.5	39.1	0.70	16	0.78	51.40
M1.05-1.9-1	200	150	167	0.0120	250	12.5	39.4	1.05	16	0.78	142.30
M1.05-1.9-2	200	150	167	0.0120	290	12.5	39.4	1.05	16	0.78	117.50
M1.05-1.9-3	200	150	167	0.0120	330	12.5	39.4	1.05	16	0.78	97.40
M1.05-1.9-4	200	150	167	0.0120	370	12.5	39.4	1.05	16	0.78	63.60
M1.05-1.9-5	200	150	167	0.0120	410	12.5	39.4	1.05	16	0.78	58.90
M1.05-1.9-6	200	150	167	0.0120	450	12.5	39.4	1.05	16	0.78	56.40
Adam, Said and Ekkarib (2016)											
B2	150	450	425	0.0035	425	10	27	0.50	50	1.00	164.65
B3	150	450	425	0.0035	425	10	27	0.75	50	1.00	174.25
B4	150	450	425	0.0035	425	10	27	1.00	50	1.00	93.75
B7	150	450	425	0.0035	425	10	27	1.00	50	1.00	199.50
B8	150	450	425	0.0035	425	10	27	1.00	50	1.00	222.60
B9	150	450	425	0.0035	340	10	27	1.00	50	1.00	257.20
B10	150	450	416.57	0.0036	250	10	27	1.00	50	1.00	298.85
B11	150	450	425	0.0035	425	10	27	1.00	50	1.00	151.20
B12	150	450	425	0.0035	425	10	27	1.00	50	1.00	220.85
Praveen and Rao (2019)											
SFRSCC30-0	100	200	180	0.0126	360	20	48.76	0.50	30	0.50	42.12
SFRSCC70-0	100	200	180	0.0126	360	20	86.66	0.50	30	0.50	45.90
Rawashdeh (2015)											
S28-VF1	120	220	178	0.0588	700	10	34.52	0.40	35	0.55	63.30
S28-VF2	120	220	178	0.0588	700	10	34.52	0.80	35	0.55	78.30
S28-VF3	120	220	178	0.0588	700	10	34.52	1.20	35	0.55	120.50
D28-VF1	120	220	178	0.0588	400	10	34.52	0.40	35	0.55	78.80
D28-VF2	120	220	178	0.0588	400	10	34.52	0.80	35	0.55	146.20
D28-VF3	120	220	178	0.0588	400	10	34.52	1.20	35	0.55	134.50
S60-VF1	120	220	178	0.0588	700	10	61.7	0.40	35	0.55	122.30
S60-VF2	120	220	178	0.0588	700	10	61.7	0.80	35	0.55	123.60
S60-VF3	120	220	178	0.0588	700	10	61.7	1.20	35	0.55	106.60
D60-VF1	120	220	178	0.0588	400	10	61.7	0.40	35	0.55	115.70
D60-VF2	120	220	178	0.0588	400	10	61.7	0.80	35	0.55	132.00
D60-VF3	120	220	178	0.0588	400	10	61.7	1.20	35	0.55	149.20

Table A – Database of experimental results from literature of SFRC beams with longitudinal reinforcement without stirrups failing in shear (cont.)

Beam	b (mm)	h (mm)	d (mm)	ρ_1	a (mm)	d_a (mm)	f_c (MPa)	V_f (%)	L_f (mm)	D_f (mm)	V_u (kN)
Rawashdeh (2015)											
S100-VF1	120	220	178	0.0588	700	10	95.14	0.40	35	0.55	84.00
S100-VF2	120	220	178	0.0588	700	10	95.14	0.80	35	0.55	101.80
S100-VF3	120	220	178	0.0588	700	10	95.14	1.20	35	0.55	126.20
D100-VF1	120	220	178	0.0588	400	10	95.14	0.40	35	0.55	140.90
Kannam *et al*. (2018)											
SFRSCC30-360 a	100	200	180	0.0126	360	20	48.76	0.50	30	0.50	51.18
SFRSCC30-360 b	100	200	180	0.0126	360	20	48.76	0.50	30	0.50	61.29
SFRSCC70-360 a	100	200	180	0.0286	360	20	86.66	0.50	30	0.50	69.42
SFRSCC70-360 b	100	200	180	0.0286	360	20	86.66	0.50	30	0.50	79.40
Cuenca, Echegaray-Oviedo and Serna (2015)											
H-45/50BN	90	350	333.33	0.0343	1000	12	84.88	1.57	50	1.11	69.00
H-65/40BN	90	350	333.33	0.0343	1000	12	92.22	1.57	40	0.62	65.00
H-80/50BN	90	350	333.33	0.0343	1000	12	96.34	1.57	50	0.63	63.50
H-80/30BP	90	350	333.33	0.0343	1000	12	83.6	1.57	30	0.38	94.00
H-80/40BP	90	350	333.33	0.0343	1000	12	91.14	1.57	40	0.50	112.50
M-45/50BN	90	350	333.33	0.0343	1000	12	51.03	1.57	50	1.11	67.50
M-65/40BN	90	350	333.33	0.0343	1000	12	45.3	1.57	40	0.62	55.00
M-80/50BN	90	350	333.33	0.0343	1000	12	39.58	1.57	50	0.63	50.00
M-80/30BP	90	350	333.33	0.0343	1000	12	49.67	1.57	30	0.38	92.50
M-80/40BP	90	350	333.33	0.0343	1000	12	42.98	1.57	40	0.50	80.00

Source: The authors.

Nomenclature

a	shear span, distance between left of loading plate and left of support
a_v	clear shear span, distance between face of loading plate and face of support
A_t	observed value (measured) for the RMSE evaluation
A_{ct}^{f}	effective area b × d, with d limited to 150 cm
b	web width
b_f	effective width of the flange in T beams
$C_{Rd,c}$	calibration factor for the design shear capacity
d	effective depth
d_f	fiber's bonding factor
D_f	fiber's diameter
d_a	maximum aggregate size
e	factor to take effect of shear span to depth ratio into account
F	fiber factor
F_t	calculated value to the evaluation of RMSE
f_c	specified concrete compressive strength
f_{ctk}	characteristic tensile strength of concrete
f_{ck}	characteristic compressive strength of concrete
$f_{c,cubo}$	average measured concrete cube compressive strength
f_{sp}	splitting tensile strength of fiber reinforced concrete
$f_{cfIk,L2}^{f}$	characteristic value of post-cracking flexural strength for a deflection of 3.5 mm
$f_{ctR,u}^{f}$	uniaxial tensile strength of SFRC
$f_{Rk,4}$	characteristic residual flexural strength for the ultimate limit state at a CMOD of 3.5 mm
f_{Ftuk}	characteristic value of post-cracking strength for ultimate crack opening
h	beam's hight
h_f	height of flange in T beams
k'	for Sharma (1986), fator that takes into account tension tests
k	for code-based equations, size effect factor
k_f	factor that considers the contribution of flanges in T-sections (= 1 for rectangular sections)

k_F^f	factor that considers the orientation of the fibers
k_G^f	size factor, which accounts for the fact that fibers are better distributed in larger elements
L_f	fiber's length (mm)
n	parameter for effect of geometry of flanged sections
n_o	(RSME) number of observations
v_b	shear strength attributed to fibers
$V_{Rd,c}$	design shear capacity of the concrete contribution
$V_{Rd,f}$	design shear capacity of the steel fiber contribution
V_f	fiber volume fraction
V_r	design shear capacity
V_{min}	ower bound to the shear capacity defined by CNR-DT (2007)
V_u	ultimate shear capacity
z	internal lever arm, calculated as 0.9 d
$\propto_c^f$	factor that accounts for the long term effects
β	load reduction load reduction coefficient relative to the clear shear span
γ_{cf}	concrete material factor, notation used in French guideline
γ_E	additional safety factor
θ	angle of compression strut
ρ	reinforcement ratio
τ	bond strength between fibers and matrix
τ_{fd}	design value of bond strength between fibers and matrix
Ψ	aggregate's size effect factor
$\sigma_{Rd,f}$	residual tensile strength of fiber reinforced cross-section
ω	reinforcement ratio that includes the effect of fibers

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