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Volatility of intraday financial data: Multiscale Ibovespa behavior under to the COVID-19 pandemic

Volatilidade de dados intradiários: comportamento multiescala do Ibovespa frente à pandemia COVID-19

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Abstract

In financial markets, volatility modeling has been a strategy widely used because it reflects uncertainties about changes in asset prices. Incorporating peculiarities of financial series, this study estimated the volatility for the intraday index of the Brazilian stock market (Ibovespa) using ARIMA-APARCH models in different time frequencies with the aid of the *wavelet* MODWT decomposition technique. This work proposes an analysis of the impacts of the frequency components on the behavior of the volatility of intraday returns using the series of details *wavelet* in different time horizons, in an atypical period in the global financial markets, generated by the COVID-19 pandemic. The empirical results suggest low unconditional volatility and strong signs of persistence in all analyzed frequencies. The asymmetry in volatility is evidenced in the higher frequencies, the leverage effect being present only in the series of details with variations of 15-120 min., which is corroborated with the results obtained with the reconstructed series. The evidenced behaviors have an impact on the elaboration of short-term investment strategies and risk management, since the positive and negative shocks, such as those given by the world pandemic of COVID-19, have different impacts on the volatility of returns in shorter periods. The information obtained can contribute to the analysis of future atypical events in the Brazilian stock market, supporting the decision-making of economic agents.

Keywords: Volatility. Ibovespa index. ARIMA-APARCH models. *Wavelet* transform.

Resumo

Em mercados financeiros, a modelagem da volatilidade vem sendo uma estratégia muito utilizada por refletir as incertezas sobre as variações dos preços dos ativos. Incorporando peculiaridades de séries financeiras, este estudo estimou a volatilidade para o índice intradiário do mercado acionário brasileiro (Ibovespa) por meio de modelos ARIMA-APARCH em diferentes frequências temporais com o auxílio da técnica de decomposição *wavelet* MODWT. Este trabalho propõe a análise dos impactos dos componentes de frequência no comportamento da volatilidade de retornos intradiários com o uso de séries de detalhes *wavelet* em diferentes horizontes temporais, em um período atípico nos mercados financeiros mundiais, gerado pela pandemia do COVID-19. Os resultados empíricos sugerem baixa volatilidade incondicional e fortes sinais de persistência em todas as frequências analisadas. A assimetria na volatilidade é evidenciada nas frequências maiores, com efeito alavancagem presente apenas nas séries de detalhes com variações de 15-120 min., o que é corroborado com os resultados obtidos com a série reconstruída. Os comportamentos evidenciados impactam na elaboração de estratégias de investimento de curto prazo e gerenciamento de risco, uma vez que os choques positivos e negativos, como os dados pela pandemia mundial do COVID-19, têm impactos diferenciados sobre a volatilidade dos retornos em prazos menores. As informações obtidas podem contribuir na análise de futuros eventos atípicos no mercado acionário brasileiro embasando a tomada de decisão dos agentes econômicos.

Palavras-chave: Volatilidade. Índice Ibovespa. Modelos ARIMA-APARCH. Transformada de *Wavelets*.

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Introduction

The analysis of intraday financial market series presents specific challenges. These data include high-frequency components that vary over time and are characterized by complex dynamics that display frequent asset price movements during a given trading period (GALLEGATI; SEMMLER, 2014; IN; KIM, 2013). Financial returns present stylized characteristics, such as:

i) a high kurtosis value;

ii) high volatility clustered over time;

iii) the presence of seasonal patterns during asset trading and others *stylized facts* (MORETTIN, 2017).

Volatility is one of the most significant issues in the field of finance. This measure corresponds to the conditional standard deviation of asset returns, which can manifest in different ways in a financial series and which reflect uncertainties regarding price changes. The greater the variation in the price of an asset over a time period is, the greater the volatility. Certain aspects of market volatility, such as the presence of persistence and the asymmetry of information shocks in financial returns series, have been a significant focus of the empirical studies presented in Audrino and Hu (2016), Baur and Dimpfl (2018), Pan and Liu (2018), Patton and Sheppard (2015), Ramzan, Ramzan and Zahid (2012), among others.

The intrinsic complexity of these systems requires methodologies that incorporate these common characteristics of intraday financial series into volatility modeling. Thus, the ARCH (*Autoregressive Conditional Heteroskedasticity*) family of conditional heteroscedasticity models, introduced by Engle (1982), has proven important in the areas of asset pricing, portfolio selection and risk assessment as these tools address issues such as asymmetric distribution, persistence and the leverage effect, see Black (1976).

This ARCH family of models includes the APARCH (*Asymmetric Power ARCH*) model, developed by Ding, Granger and Engle (1993). In addition to addressing persistence, this model provides flexibility by incorporating an exponent of conditional standard deviation that varies with an asymmetry coefficient to consider the leverage effect resulting from the risk aversion inherent to economic agents. According to Daly (2008), the phenomenon of leverage occurs when the volatility of returns increases when the prices goes down, while volatility is less intense during periods of high prices. The APARCH also includes seven other ARCH extensions as special cases: the classic ARCH and GARCH, GJR-GARCH, TS-GARCH, TARCH, NARCH and Log-ARCH (or MGARCH).

The volatility data can be analyzed through the decomposition of time scales. Hasbrouck (2016) and Nava, Matteo and Aste (1989) demonstrate that the consideration of different time scales helps to explain the inherent complexity of financial index volatility behaviors and captures the effects of intraday and long-term patterns on price movements (LATIF *et al*., 2011; ROSSI, 2015) as well as the impacts of long- and short-term trends (GALLEGATI; SEMMLER, 2014). As a result, the use of wavelet techniques has gained many supporters (BIAGE, 2019; JENSEN; WHITCHER, 2014; OMANE-ADJEPONG; ABABIO; ALAGIDEDE, 2019; SHAH; TALI; FAROOQ, 2018).

This study examined the influence of the financial microstructure on volatility by analyzing the impacts of different time frequencies on intraday volatility behaviors. To this end, the ARIMA-APARCH process was used to estimate conditional variance at various frequencies based on the wavelet decomposition of the intraday time series of returns. This technique enabled us to capture different levels of non aggregated detail in the original series over time. The aim was to analyze, at each frequency, the effects of asymmetry, unconditional variance, the presence of persistence and the leverage effect on world financial markets during the COVID-19 pandemic.

The analysis used the intraday returns of the Ibovespa index during a period that reflects the impacts of the COVID-19 pandemic. The Ibovespa is an index of the average performance indicators of stocks traded on the Brazilian capital market, one of Latin America’s most important markets. Thus, the contribution of this study is its evaluation of the impacts of various financial cycles on estimates of the conditional variance of the Brazilian stock market, which demonstrates how prominent securities respond to the effects of volatility over different time horizons. Since this financial market has increased susceptibility to crises, and considering the sampling period of this study, its results can be used during future periods of uncertainty to support the investment management decisions of economic agents.

The period of analysis was from March 17 to September 11, 2020, with a sampling interval of 15 minutes for the seven hours of daily trading in the Brazilian market. The Maximal Overlap Discrete Wavelet Transform (MODWT) developed by Percival and Walden (2000) was used; it enables the filtering of series into different frequency components and preserves the variation in the original series with location invariance. The decomposition was performed using the Daubechies wavelet filter (DAUBECHIES, 1992) with two null moments since it exhibits important properties in the multiresolution analysis.

Methodological framework and procedures

The basic methodological structure for conditional heteroscedastic processes and the APARCH model used in this study are presented below, along with the characteristics of the MODWT decomposition. These procedures were performed with the assistance of *software R* (R CORE TEAM, 2020) using the *fGarch* (WUERTZ *et al.*, 2019) and *wmtsa* (CONSTANTINE; PERCIVAL, 2017) software packages.

Data

Ibovespa index quote data were obtained using the *QuantTools* (KOVALEVSKY, 2018) package. Each record contains information about price, trade volumes and trade date and time. Table 1 shows the composition of the Ibovespa index. The analysis period was March 17 to September 11, 2020, and $n = 3660$ observations are included. The sampling interval was every 15 minutes for the seven hours of continuous trading in the Brazilian stock market. The analyzed series corresponds to the intraday log-returns $r_{d,m} = \ln(p_{d,m}) - \ln(p_{d,m-1})$, where $p_{d,m}$ is the asset price in period $m = 1,\ldots,28$ of business day $d = 1,\ldots,124$. The notation r_t used here represents the intraday series of returns of the Ibovespa $r_{d,m}$.

Wavelet analysis

The MODWT filters a time series into multiscale information. The MODWT exhibits the essential properties of a time series decomposition: it is non-orthogonal and location-invariant, thus preserving variation in the original series, for more details, see Percival and Walden (2000). These characteristics enable the estimation of volatility using reconstructed series of decomposition.

Table 1 – Main stocks comprising the Ibovespa index from March to December 2020.

S/A	Storage code	Participation
Vale do Rio Doce	VALE3	10.46%
Itaú Unibanco	ITUB4	6.38%
Petrobras	PETR4	5.62%
B3	B3SA3	5.33%
Bradesco	BBDC4	5.09%
Petrobras	PETR3	4.40%
Magazine Luiza	MGLU3	3.19%
Ambev	ABEV3	2.95%
Banco do Brasil	BBAS3	2.34%
Weg	WEGE3	2.34%
Intermédica	GNDI3	2.33%
Lojas Renner	LREN3	1.95%
Natura	NTCO3	1.91%
Suzano	SUZB3	1.85%
JBS	JBSS3	1.84%

Source: The authors.

To generate the wavelet financial series, iterative filtering of the returns data is needed. For all integers n of length L, the low-pass $g_{j,l}$ and high-pass $h_{j,l}$ filters are applied to decompose signal and must meet the following criteria:

i) $\sum_{l=0}^{L-1} h_l = 0$;

ii) $\sum_{l=0}^{L-1} h_l^2 = \frac{1}{2}$;

iii) $\sum_{i=0}^{L-1} h_i h_{i+2n} = 0$;

iv) $g_l = -1^l h_{l-}$.

The MODWT $\{g_{j,l}\}$ and $\{h_{j,l}\}$ dimension filters are expressed as follows: $\widetilde{h}_{j,l} = h_{j,l}/2^j$ and $\widetilde{g}_{j,l} = g_{j,l}/2^j$. Because of the advantages offered by compact support and orthogonality, this study uses the *Daubechie wavelet* $h_{l,j} = (-1)^{l-L_j} g_{L_j-1-l}$ with two null moments ($db2$) (DAUBECHIES, 1992).

Using $\widetilde{s}_{0,t} = r_{t=0}^{N-1}$ as the initial input, the decomposition process is performed using the pyramidal algorithm developed by Mallat (1989) from the relationships presented in equations (1) and (2)

$$\widetilde{s}_{j,t} \equiv \sum_{l=0}^{L_j-1} \widetilde{g}_{j,l} r_{t-l \bmod N} \quad (1)$$

and

$$\widetilde{d}_{j,t} \equiv \sum_{l=0}^{L_j-1} \widetilde{h}_{j,l} r_{t-l \bmod N}, \quad (2)$$

in which $L_j = (2^j - 1)(L-1) + 1$ corresponds to the size of the filter associated with each scale j, $j = 1, \ldots, J$, and $\mathrm{mod}\, N$ is a module operator. At j levels and time period t, the coefficients of scale $\widetilde{s}_{j,t}$ refer to the r_t approach, which captures longer fluctuations, and coefficients $\widetilde{d}_{j,t}$ capture fluctuations as structural changes, representing the r_t details. Since the MODWT preserves the original variation of the input series, and given the relationships presented in equations (1) and (2), a reconstruction of r_t can be obtained, as shown in equation (3)

$$r_t = \sum_{j=1}^{J} \widetilde{d}_{j,t} + \widetilde{s}_{J,t}. \tag{3}$$

Details wavelet series

The $\widetilde{d}_{j,t}$ coefficients are the objects of our analysis as they consist of time series that describe r_t in increasingly coarse levels of resolution that are not aggregated over time. Each level of detail corresponds to a cycle. The higher the level of decomposition is, the longer the time interval of the cycle (CROWLEY, 2007). The $\widetilde{d}_{j,t}$ series uses approximate capture variations in minutes, hours, weeks and months. The frequencies are measured according Crowley (2007) by Table 2, and are based on the Brazilian market workday.

Table 2 – Variation in and interpretation of the decomposition components of the Ibovespa intraday log-returns

Level	Frequency
$\widetilde{d}_{1,t}$	15-30 trading minutes
$\widetilde{d}_{3,t}$	60-120 trading minutes
$\widetilde{d}_{6,t}$	8-16 hours $\approx$ 1-2 trading days
$\widetilde{d}_{8,t}$	32-64 hours $\approx$ 5-9 trading days
$\widetilde{d}_{10,t}$	128-256 hours $\approx$ 18-37 trading days

Source: The authors.

Because the weekly variations captured by $\widetilde{d}_{8,t}$ are influenced by the days of the week (the so-called Monday and Friday effects), to adjust for volatility at this level it is necessary to isolate this seasonality as Alberg, Shalit and Yosef (2008), according to the equations (4) and (5), given by

$$\widetilde{d}_{8,t} = \alpha_1 \mathrm{Mo}_t + \alpha_2 \mathrm{Tu}_t + \alpha_3 \mathrm{We}_t + \alpha_4 \mathrm{Th}_t + \alpha_5 \mathrm{Fr}_t + \delta_t \tag{4}$$

and

$$(\widetilde{d}_{8,t} - \hat{\widetilde{d}}_{8,t})^2 = \alpha_1 \mathrm{Mo}_t + \alpha_2 \mathrm{Tu}_t + \alpha_3 \mathrm{We}_t + \alpha_4 \mathrm{Th}_t + \alpha_5 \mathrm{Fr}_t + \varepsilon_t \tag{5}$$

for which Mo_t, Tu_t, We_t, Th_t, Fr_t, are *dummy* variables representing the days of the week, and $\hat{\widetilde{d}}_{8,t}$ is the value predicted by equation (4). The filtered series is obtained from equation (5) is $\widetilde{d}_{8,t} = \frac{(\widetilde{d}_{8,t} - \hat{\widetilde{d}}_{8,t})}{\sqrt{\hat{\eta}}}$, for which $\hat{\eta}$ is the estimated value of $(\widetilde{d}_{8,t} - \hat{\widetilde{d}}_{8,t})^2$.

APARCH model

The ARCH group of processes generally expresses the volatility component (conditional variance) and an innovation component commonly assumed to be Gaussian,t-*Student* or an extension of these distributions. Likewise, the APARCH model was developed to introduce the flexibility of a variable exponent with an asymmetry coefficient to detect the asymmetric impacts of shocks on the volatility (leverage effect) of a series of financial returns.

The general structure of the APARCH(p,q) model for a conditional standard deviation (σ_t) of r_t is given by $r_t = \mu + \varepsilon_t$, in which $\varepsilon_t = z_t \sigma_t$ and $z_t \sim \mathscr{D}_\nu(0,1)$, with σ_t^δ presented in equation (6)

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \tag{6}$$

subject to the following parameter restrictions to ensure that the conditional variance is positive and weakly stationary:

- $\omega \geq 0$, $\delta \geq 0$, $|\gamma_i| \leq 1$;
- $\alpha_i \geq 0$, $i = 1, \ldots, p$, $\beta_j \geq 0$, $j = 1, \ldots, q$;
- $0 \leq \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_i \leq 1$, otherwise.

The ω parameter corresponds to the average degree of volatility of the conditional variance. The δ parameter enables the estimation of other powers to determine the conditional standard deviation using a Box-Cox transformation in σ_t. The α_i coefficient indicates the reaction of the volatility to a shock in the series, and β_j measures the amount of volatility from the previous period that persists in the current period.

The sum of α_i and β_i represents the persistence of volatility, subject to the restriction given above. If case $\sum(\alpha_i + \beta_j) > 1$, the series shows a high persistence of volatility, i.e., volatility shocks will last for a very long time in the series. The presence of the leverage effect is indicated if $\gamma_i > 0$. The term $\mathscr{D}_\nu$ detects, if present, a distribution with a heavy tail and indicates an error distribution with mean 0 and variance 1, where the distribution is normal if $\nu = 0$.

The μ conditional average is modeled by using an autoregressive integrated moving average (ARIMA) model and applying the Box and Jenkins (1970) methodology. ARMA(p,q) denotes an autoregressive and moving average model of order (p,q) that can be specified as described by equation (7)

$$r_t = \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t, \tag{7}$$

in which the ε_t term is the residual that comprises independent and identically distributed random variables (iid) with zero mean and variance σ^2, $\phi_i, \theta_j \in \mathbb{R}$, with $i = 1,\ldots,p$ and $j = 1,\ldots,q$.

From the ARMA model definition, the hypothesis of the stationary time series provides an analogous ARIMA$(p,0,d)$ process; otherwise, r_t must be differentiated, and the ARIMA(p,d,q) must be adjusted, with d as the order of differentiation of the series. The stationarity analysis of each series is conducted using the *Augmented Dickey-Fuller* (ADF) test developed by Dickey and Fuller (1979, 1981).

With series data representing r_t at different j frequencies, ARIMA-APARCH models are estimated with $\widetilde{d}_{j,t}$, using a maximum likelihood (ML) approach. To model the conditional variance after the ARIMA(p,d,q) model is identified and estimated, it is necessary to make an additional assumption regarding the density function $\mathscr{D}_\nu$, denoted by $g(z_t;\tau)$, where τ is a vector of distribution parameters. The log-likelihood function is given by equation (8)

$$l(\theta,\tau) = \sum_{t=1}^{T} \log f_j\left(\tilde{d}_{j,t} \mid \theta,\tau,I_{t-1}\right), \tag{8}$$

with the maximum likelihood estimator (MLE) of

$$\hat{\theta}_{hMLE} = \max_{\theta\in\Theta} l(\theta,\tau),$$

in which θ is the vector of unknown parameters for the conditional mean and variance, and I_{t-1} are the data defined in period t.

Since financial time series often exhibit non-normal distributions with excess kurtosis and asymmetry, a more appropriate distribution for z_t is the t-*Skewed*(ν,ξ), in which the parameters ν and ξ represent the degrees of freedom and asymmetry, respectively. For the GARCH structure, Lambert and Laurent (2001) recommend using the function $l(\theta,\tau)$, given t-*Skewed* distribution, described by the equation (9)

$$\begin{aligned} l(\theta,\nu,\xi) = {} & \mathrm{T}\left[\log\Gamma\left(\frac{\nu+1}{2}\right) - \log\left(\frac{\nu}{2}\right) - \frac{1}{2}\log(\pi(\nu-2))\right] \\ & + \mathrm{T}\left[\log\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + \log(\mathrm{s})\right] - \frac{1}{2}\sum_{t=1}^{T}\left[\log\left(\sigma_{\mathrm{t}}^2\right)\right] \\ & - \frac{1}{2}\sum_{t=1}^{T}\left[(1+\nu)\log\left(1+\frac{(sz_t+m)^2}{\nu-2}\xi^{-2I_t}\right)\right], \end{aligned} \tag{9}$$

in which

$$I_t = \begin{cases} 1, & \text{if} \quad z_t \geq -\frac{m}{s}, \\ -1 & \text{if} \quad z_t < -\frac{m}{s}, \end{cases}$$

$$m = \frac{\Gamma((\nu+1/2))\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\nu/2)}\left(\xi - \frac{1}{\xi}\right)$$

and

$$s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}.$$

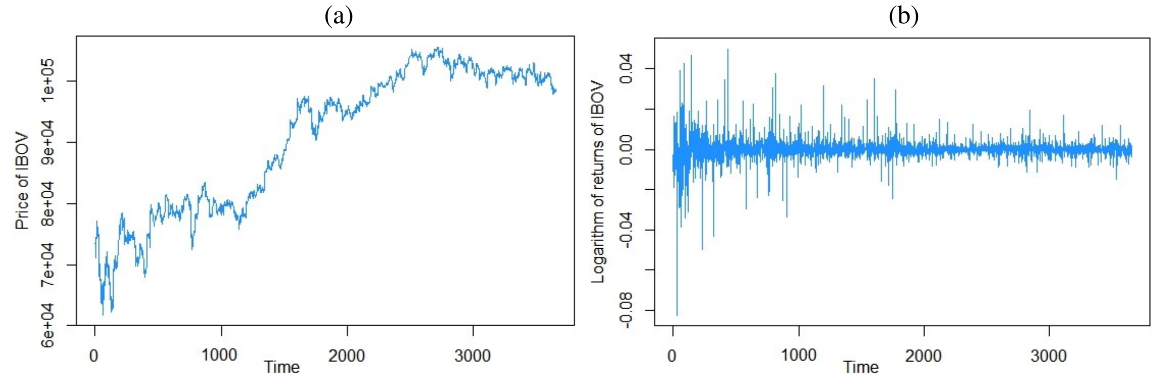
Results and discussion

Data and descriptive statistics

Figure 1(a) shows graphs of prices and Figure 1(b) of the intraday log-returns (Δ = 15 min) from March 17 to September 11, 2020. The price movements clearly reflect the intraday behavior and the log-returns volatility groupings. These behaviors reflect the impacts of the COVID-19 pandemic on world financial markets, which experienced an initial shock in March 2020. In addition to the pandemic, the influence of instability in Brazilian politics can be observed during this period. It is also necessary to consider the increasing investment in the Brazilian capital market. The descriptive analysis exhibits peculiarities such as negative asymmetry with a value of -0.65, and an excess of kurtosis equal to 40.14. The *Kolmogorov-Smirnov* test produced a KS = 0.49, which is also not normal.

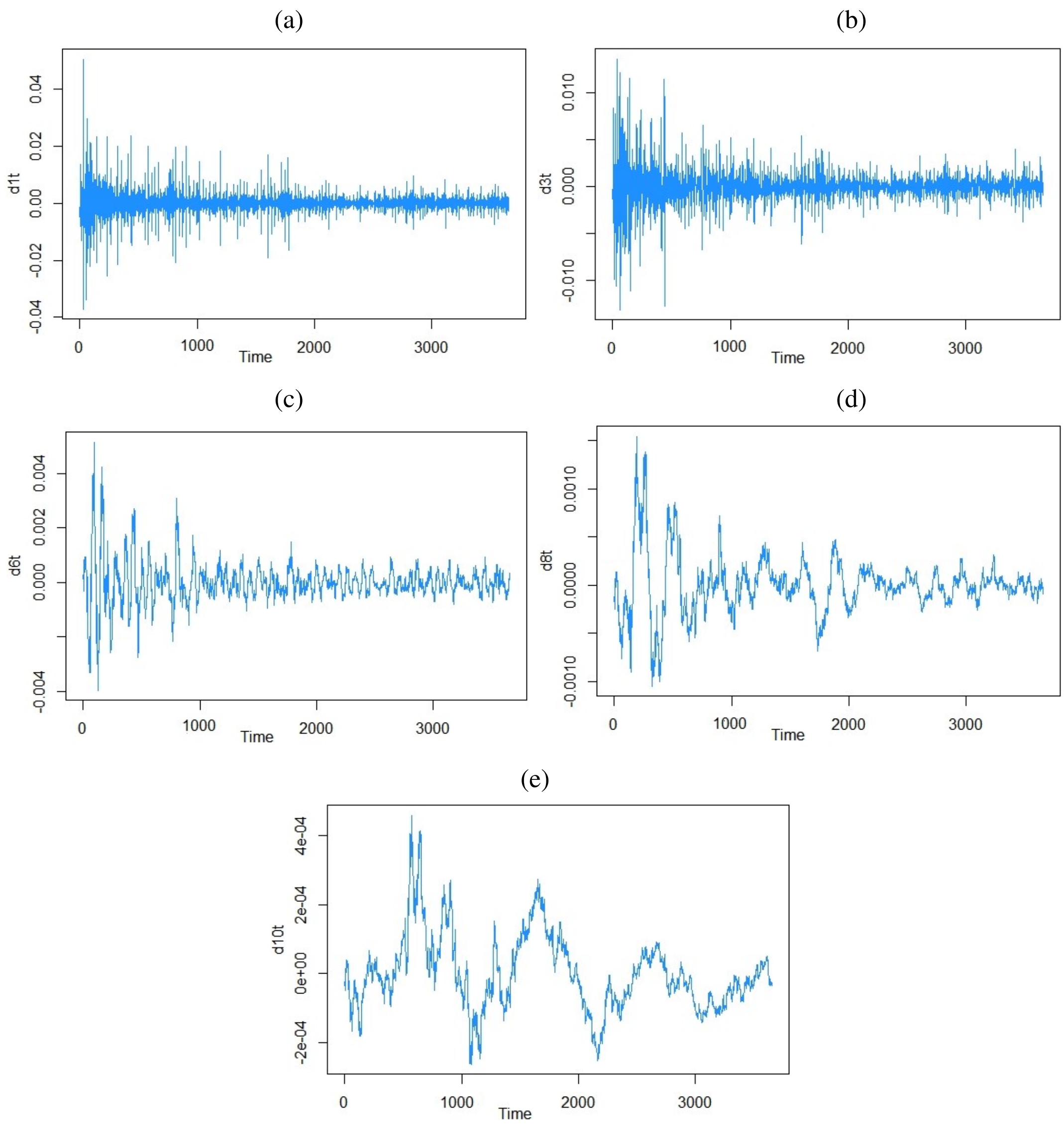
The MODWT decomposition process was applied to the series using the methodology described in the previous section to obtain $J = 11$ levels of decomposition. For the APARCH adjustment, we used the detail *wavelet* series $\widetilde{d}_{j,t}$, $j = 1,3,6,8,10$, corresponding to the variation frequencies from the very short term (stochastic noise) to the medium and long terms. The series obtained are shown in Figures 2(a)-2(e).

Figure 1 – Ibovespa series from March 17 to September 11 ($\Delta = 15$min): (a) Price series; (b) Series of intraday log-returns.



Source: The authors.

Figure 2 – Series $\widetilde{d}_{j,t}$, with $j = 1, 3, 6, 8, 10$, or the MODWT decomposition of the intraday Ibovespa log-returns from March 17 to September 11 (Δ =15 min).



Source: The authors.

Series $\widetilde{d}_{8,t}$ captures the Friday effect. Thus, the filtered series was used to model the volatility according to the methodology presented above, which is based on the estimations of equations (3) and (4). The results of the regression, with the standard errors in parentheses, are shown in Table 3.

Table 3 – Regression coefficients for the day of the week effect for $d_{8,t}$.

Coefficients	Estimates
α_1	0.000063 (0.00001)
α_2	0.000016 (0.00001)
α_3	−0.00000014 (0.00001)
α_4	0.000017 (0.00001)
α_5	−0.000039* (0,00001)

*significant at 5%

Source: The authors.

Volatility analysis

Once the series obtained by the wavelets coefficients were defined, the stationarity condition was analyzed. As shown in Table 4, the ADF test results without constant (no ct) and with constant and trend (ct+trend) confirmed that $\widetilde{d}_{10,t}$ is the only non stationary series.

Table 4 – Unit root ADF test for details series $\widetilde{d}_{j,t}$, $j = 1,3,6,8,10$.

Series	Option	p-Value
$\widetilde{d}_{1,t}$	no ct	≤ 0.01
	ct+trend	≤ 0.01
$\widetilde{d}_{3,t}$	no ct	≤ 0.01
	ct+trend	≤ 0.01
$\widetilde{d}_{3,t}$	no ct	≤ 0.01
	ct+trend	≤ 0.01
$\widetilde{d}_{6,t}$	no ct	≤ 0.01
	ct+trend	≤ 0.01
$\widetilde{d}_{8,t}$	no ct	≤ 0.01
	ct+trend	≤ 0.01
$\widetilde{d}_{10,t}$	no ct	≤ 0.01
	ct+trend	0.04

Source: The authors.

The ARIMA(p,d,q)-APARCH$(1,1)$ processes were estimated next. The specifications for the conditional average were given by ARIMA$(1,0,1)$, ARIMA$(0,0,8)$, ARIMA$(2,0,0)$, ARIMA$(1,0,0)$ and ARIMA$(2,1,1)$ models for $\widetilde{d}_{1,t}$, $\widetilde{d}_{3,t}$, $\widetilde{d}_{6,t}$, $\widetilde{d}_{8,t}$ filtered and $\widetilde{d}_{10,t}$, respectively. The results of the parameter estimates are shown in Table 5, with the standard errors in parentheses. The *Box-Pierce* $Q(20)$ statistics for the standardized residuals indicate the goodness of fit of the models.

Regarding the results for μ with the estimated values of the explanatory variables defined by ϕ and θ, it should be noted that for the frequency represented by series $\widetilde{d}_{3,t}$, Figure 2(b), the results reflect the persistence of price movements during the day, as was also observed by Schulmeister (2009). It was necessary to adjust ARIMA$(p,1,q)$ for $\widetilde{d}_{10,t}$, Figure 2(e), as this level represents the lower frequency movements in the medium- to long-term trend, which demonstrates the dynamism of asset trading activities in the financial market. This behavior is substantiated by the ADF test results and by the correlogram presented in Figure 3, which shows the autocorrelation values slowly trending downwards to zero.

The results for σ show low unconditional volatility on every scale, as given by the ω estimates. The ξ asymmetry parameter was significant and positive, and the ν estimate captured the presence of heavy tails. The α_1 and β_1 coefficients were also significant for every series. The asymmetry and leverage effects on volatility were observed only in the decomposition of short-term oscillations and stochastic noise.

The low α_1 values for every frequency except the weekly frequency indicate that β_1 the effects of shocks on volatility are felt quickly. The shocks experienced during the examined period mainly correspond to uncertainties in the global economy pertaining to the COVID-19 pandemic. For the intraday, daily and monthly frequencies, more than 90% of the shock in the $t-1$ time series persisted at time t. For the weekly frequency, 78% of the shock persisted. Furthermore, for all frequencies in which $\sum\alpha_1 + \sum\beta_1 > 1$, the effects of shocks on volatility persisted for a longer period in the series. In other words, the process of reversing the conditional variance to its mean value tends to be slow after a shock.

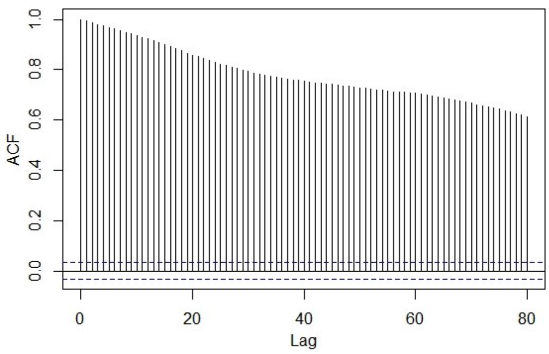
The leverage effect represented by $\gamma_1 > 0$ was only exhibited at the $\widetilde{d}_{1,t}$ and $\widetilde{d}_{3,t}$ levels, indicating that past negative shocks, such as the effects of COVID-19, economic and political instabilities and exogenous changes in transaction volumes, have a more intense impact on t time-conditional volatility than past positive shocks. The presence of asymmetry in $\widetilde{d}_{6,t}$, $\gamma_1 < 0$, indicates that positive shocks have a greater impact on volatility than negative ones. For the other series, γ_1 was not significant, indicating that price drops and hikes could have the same effect on volatility.

Table 5 – ARIMA-APARCH model estimates for series $\widetilde{d}_{j,t}$. $j = 1,3,6,8,10$

Coefficients	$\widetilde{d}_{1,t}$	$\widetilde{d}_{3,t}$	$\widetilde{d}_{6,t}$	$\widetilde{d}_{8,t}$	$\widetilde{d}_{10,t}$	Coefficients	$\widetilde{d}_{1,t}$	$\widetilde{d}_{3,t}$	$\widetilde{d}_{6,t}$	$\widetilde{d}_{8,t}$	$\widetilde{d}_{10,t}$
ϕ_1	−0.02** (0.01)		0.99* (0.01)	0.98* (0.00)	0.30* (0.02)	ω	5e^{-5}* (0.00)	1e^{-6}* (0.00)	3e^{-8}* (0.00)	7e^{-4}* (0.00)	6e^{-8}* (0.00)
ϕ_2			−0.03* (0.01)		−0.02** (0.00)	α_1	0.10* (0.02)	0.11* (0.00)	0.05* (0.01)	0.56* (0.07)	0.04* (0.00)
θ_1	−0.99* (0.00)	0.95* (0.00)			−0.29* (0.02)	γ_1	0.31* (0.08)	0.28* (0.07)	−1.32** (0.07)	−0.06 (0.05)	−1.21 (0.11)
θ_2		0.89* (0.00)				β_1	0.93* (0.014)	0.91* (0.01)	0.95* (0.00)	0.78* (0.01)	0.96* (0.00)
θ_3		0.85* (0.00)				δ	1.18* (0.12)	1.25* (0.13)	1.38* (0.17)	0.35* (0.03)	0.98* (0.25)
θ_4		−1.00* (0.00)				ξ	1.01* (0.01)	1.01* (0.01)	1.00* (0.01)	0.99* (0.01)	1.00* (0.01)
θ_5		−0.92* (0.00)				ν	2.50* (0.11)	2.85* (0.15)	3.26* (0.18)	2.01* (0.01)	2.91* (0.15)
θ_6		−0.87* (0.00)				$Q(20)$	0.55	0.12	0.30	0.99	0.38
θ_7		−0.83* (0.00)									
θ_8		0.02* (0.00)									

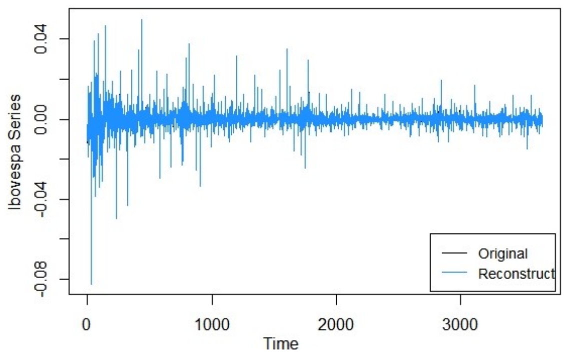
*significant at 5%, **significant at 10%
Source: The authors.

Figure 3 – Autocorrelation function for the $\widetilde{d}_{10,t}$ series, MODWT decomposition of the Ibovespa intraday logreturns from March 17 to September 11 ($\Delta = 15$min).



Source: The authors.

Figure 4 – Reconstructed series with $\widetilde{d}_{j,t}$, for $j = 1,2,3$, of the MODWT decomposition of Ibovespa intraday logreturns ($\Delta = 15$min).



Source: The authors.

The results for the γ_1 coefficient indicate that the first levels of decomposition are responsible for most of the r_t variability (around 89%), as was also observed by Biage (2019) and Kumar and Anandarao (2019).

Since decomposition of higher frequencies better enabled the detection of the leverage effect, the ARIMA(p,d,q)-APARCH$(1,1)$ processes were applied in the reconstructed $r_t = \widetilde{d}_{1,t} - \widetilde{d}_{3,t} = \sum_{j=1}^{3} \widetilde{d_{j,t}}$ series using equation (3). In Figure 4, we can observe the resulting reconstruction. The results of the ADF test showed no unit root evidence for the reconstructed series (p-value $\leq$ 0.01 for no ct and ct+trend cases). The ARIMA$(2,0,1)$-APARCH$(1,1)$ model was adjusted, and the results are shown in Table 6 (with standard errors in parentheses), which shows that asymmetry, the leverage effect and persistence were all exhibited during this period.

Table 6 – ARIMA-APARCH model estimates for the Ibovespa intraday reconstructed series ($\Delta = 15$min).

Coefficients	Ibovespa	Coefficients	Ibovespa
ϕ_1	-0.630* (0.291)	β_1	0.880* (0.022)
ϕ_2	-0.680* (0.282)	δ	1.300* (0.264)
θ_1	0.656* (0.289)	ξ	0.927* (0.026)
ω	0.006* (0.002)	ν	4.197* (0.410)
α_1	0.139* (0.025)	$Q(20)$	0.670
γ_1	0.120* (0.007)		

*significância de 5%
Source: The authors.

Conclusion

Considering the dynamics of financial assets, the main objective of this study was to determine the behavior of conditional volatility components in intraday financial series for short-, medium- and long-term cycles. To this end, we used the MODWT technique to separate the Ibovespa log-returns series (Δ = 15min) into the following five frequency components: $\widetilde{d}_{j,t}$, $j = 1, 3, 6, 8, 10$. The particular characteristics of intraday financial series, such as asymmetry, seasonality and volatility groupings, led to select ARIMA(p,d,q)-APARCH$(1,1)$ models with a t-Skewed(ν,ξ) distribution.

The results show that for the sampled period, every frequency of Ibovespa returns exhibited low unconditional volatility and asymmetry. In addition, volatility tended to persist in the series for a long time at every frequency. Asymmetric effects of shocks on volatility were significant for the 1- to 2-day frequency, but the leverage effect was only exhibited for very short frequencies (15-120 min), revealing that negative price fluctuations have a greater effect on volatility in the very short term. The results led us to conclude that the impacts of high frequency decomposition, which also had significant effects on the series of returns, exhibit a leverage effect.

The estimates produced by the modeling of the reconstructed series with the first three MODWT levels corroborate the results for the frequencies. The results of this study show that models that apply multiscale resources and *wavelet* techniques can provide period-specific information regarding intraday volatility in financial markets. During times of uncertainty, such as the current global COVID-19 pandemic, such information and its impacts on conditional variance are essential for the development of short- and long-term hedge strategies by economic agents. Therefore, the findings of this study may contribute to financial management and analysis, especially during events that increase uncertainty in the Brazilian stock market.

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