

## *Evaluation of railway track deflections using the finite element method*

### *Avaliação das deflexões da via férrea utilizando o método dos elementos finitos*

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#### **Abstract**

Rail transport is linked to the efficiency of transport infrastructure, and the increasing inefficiency of Brazil's road network has highlighted the importance of rail transport for the country. Therefore, alleviation of logistical bottlenecks has been pursued, and alternative proposals aimed at improving the railway transportation mode in Brazil have gained increasing prominence. In this sense, knowledge of railway track mechanical behavior is essential, both for maintenance purposes and for designing new railways. Thus, the goal of this study is to analyze a railway track mechanical behavior using the finite element method. A three-dimensional model of the rail was developed, and the other railway components were simulated by considering springs with constant stiffness coefficients, according to the concept of the support stiffness coefficient. The results obtained were compared to data from the literature on the subject and to analytical formulations based on the Talbot method. Similar deflection results were found in the models compared; divergences occurred due to the different considerations of the models.

**Keywords:** Railway. Finite element method. Track deflection.

#### **Resumo**

O transporte ferroviário está ligado à eficiência da infraestrutura de transportes. A sua importância ficou evidenciada com as crescentes dificuldades apresentadas pelo transporte rodoviário nacional. Por isso, tem-se buscado o alívio dos gargalos logísticos, e as propostas que visam melhorias no modal ferroviário têm ganhado cada vez mais atenção como alternativa. Nesse sentido, o conhecimento do comportamento mecânico da via é fundamental, tanto para a manutenção, quanto para o projeto de novas ferrovias. Assim, o objetivo deste trabalho é estudar o comportamento mecânico da via férrea, através de uma modelagem com o emprego do método dos elementos finitos. Para tanto, foi elaborado um modelo tridimensional do trilho, e os demais componentes da via férrea foram simulados pela consideração de molas com coeficientes de rigidez constantes, conforme o conceito do coeficiente de rigidez de apoio. Os resultados obtidos foram comparados com dados da literatura e formulações analíticas, a partir do método de Talbot. Como resultado, foram encontradas bacias de deflexão similares entre os modelos comparados, as suas divergências ocorreram em função das diferentes considerações dos modelos.

**Palavras-chave:** Via férrea. Método dos elementos finitos. Deflexão da via.

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## Introduction

According to Monteiro (2015), the rail transportation mode plays an important role in the efficiency of transport infrastructure: it is more advantageous than the road mode for cargo above 40 tons, regardless of transport distance, in addition to generating lower environmental and social costs, as asserted by the Brazilian National Confederation of Transport (CNT, 2013). Data from CNT (2018) also shows that in 2017, the Brazilian rail network comprised 30,485 km, with a growth of 45.9% in tone-kilometers (tkm) in the previous ten years.

In order to increase the use of the rail mode in the country's transport matrix, efforts have been put forward to alleviate logistical bottlenecks and expand the national rail system (CNT, 2013). An example of such a bottleneck comes from soybean cargo. Its transportation by rail guarantees scale gain, with a reduction in product cost and an increase in competitiveness. However, soybean corresponds to only 7.4% of the rail-transported cargo in Brazil since the road mode is still widely used for this purpose (CNT, 2013).

Brazil's high dependency on the road mode highlights the imbalance of the country's transport matrix, which culminated in the truckers' strike in 2018. There is current demand to improve the capacity of cargo rail transport by increasing axle load. Therefore, further understanding of rail track mechanical behavior could enhance the development of more adequate projects that consider safety issues and the deterioration of track components that would result from axle load increase (MONTEIRO, 2015).

According to Selig and Waters (1994), railway models allow for joint evaluation of superstructure and infrastructure components, making it possible to consider their complex interaction and to establish responses to stresses and deformations that are involved in the loading of railway vehicles. In addition, these models allow predictions about railway performance, assisting in making dimensional design decisions and supporting maintenance planning (SPADA, 2003).

Tolentino and Souza (2019) argue in favor of the value of computational methods for modeling, analyzing, and verifying structural data. In this way, Giner *et al.* (2016) indicates that numerical analysis, as the finite element method (FEM), is adequate to analyze complex problems, as railway mechanical analysis. According to Gallego *et al.* (2013), railway track modeling using finite element models can be an important tool for the design of this structure.

In the same vein, the present study seeks to evaluate railway mechanical behavior via finite element modeling, considering the rail track as a solid component and the other components as elastic springs, based on the concept of the support stiffness coefficient. In future studies, the model developed here can be expanded to consider the other railway elements three-dimensionally in order to bring the model's responses closer to real railway behavior.

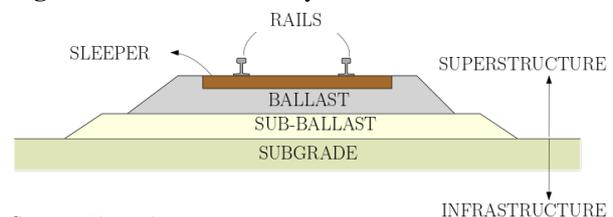
The finite element method (FEM) has been used in several studies in the analysis of railways (FERREIRA; TEIXEIRA, 2012; MATIAS, 2014; RANGEL; ARAGÃO; MOTTA, 2015; SAYEED; SHAHIN, 2016; SILVA FILHO, 2016). Here, the method was applied using Ansys® software to simulate the railway and to obtain the displacement values. The results were compared with the data from the analytical formulation, obtained through the Talbot method, and with the ones from an analysis performed using the FTool® software.

In the next sections, the conceptual framework of the present research is presented. It is followed by an outline of the methodology used and a discussion of the results obtained.

## Theoretical Framework

A railway track comprises a superstructure and an infrastructure, as shown in Figure 1. According to Medina and Motta (2015), the superstructure is composed of rails, sleepers, ballast, and sub-ballast. The infrastructure consists of all the elements that support the superstructure, such as backfill, engineering structure, among others (NABAIS, 2014).

**Figure 1 – Ballasted railway track**



Source: The authors.

The superstructure can be classified whether as rigid, when the sleepers are supported on concrete slabs or on a beam, or as elastic, when a ballast bed is used to allow a more proper distribution of loads on the track's infrastructure (NABAIS, 2014). As Figure 1 shows, the present study deals with the superstructure of a ballasted track, which is the type of superstructure most commonly used for railways (KALLIAINEN; KOLISOJA; NURMIKOLU, 2016).

In the superstructure, the rails form the rolling surface of the wheels (MEDINA; MOTTA, 2015). They guide the railway vehicles and transfer the load of the wheels to the sleepers, which display continuous beam behavior (NABAIS, 2014). Vignole rails are the most used type of rail (MEDINA; MOTTA, 2015). They consist of the rail head (the upper part, which is in contact with the wheel), the rail base (the lower part, which connects to the sleeper) and the rail web (which connects the head to the base).

According to Sarmiento (2015), the sleepers support the rails, maintaining their position and allowing their fixation, in addition to transferring loads from the railway vehicles to the ballast. Currently, sleepers can be made of wood, concrete, steel, among other materials (NABAIS, 2014), with the first two being the most commonly used (SARMENTO, 2015). Wooden sleepers distribute loads more evenly than concrete ones; however, they have a higher cost and a shorter useful life (MEDINA; MOTTA, 2015).

In ballasted railway tracks, the sleepers rest on the ballast, a layer of crushed stone that distributes the stress from the sleepers, securing flexibility to the railway (MEDINA; MOTTA, 2015). In addition, the ballast permits track geometry adjustment and helps with drainage. Furthermore, it offers resistance to the forces applied on the sleepers (SELIG; WATERS, 1994). For this reason, stone ballast is considered superior to ballast made from other materials due to its resistance and elasticity (NABAIS, 2014).

The function of the sub-ballast is to reduce the stresses to a value suitable for the foundation ground, thus allowing the ballast layer, which is a higher added value product, to be reduced (SELIG; WATERS, 1994). In addition, the sub-ballast prevents the ballast from penetrating the foundation, which contributes to the drainage capacity and elasticity of the track.

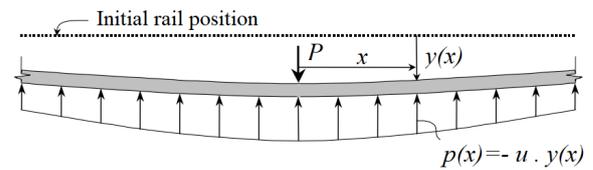
Hay (1982) posits that the proper design of the superstructure elements depends on the analysis of the track to determine load limits and their corresponding deflections, rail-bending moments and stresses, contact and shear stresses, reactions to the predicted load conditions and to the characteristics attributed to the track.

A number of theories and techniques have been developed for railway track analysis (HAY, 1982). According to Teixeira (2004), the earliest research on track behavior made in XIX century, is attributed to Winkler, who considered the track as a continuous beam, uniformly supported on an elastic base, which was defined by a ballast coefficient (C) (WINKLER, 1867).

The ballast coefficient can be defined as the stress per unit length that produces a unit deflection in the ballast and therefore has its dimension defined by the function  $[F] [L]^{-3}$ , where F is a unit of force, and L a unit of length (HAY, 1982).

At the beginning of the 20th century, Talbot (1918), working from Winkler's formulation, used the track modulus ( $u$ ) to characterize the elastic base. This parameter is the load that, once uniformly distributed on the rail, produces a unit deflection, defined as  $[F] [L]^{-2}$  (TEIXEIRA, 2004). Figure 2 below represents the model proposed by Talbot.

**Figure 2** – Track on elastic base (foundation)



**Source:** Adapted from Teixeira (2004), with authorization to reproduce the image.

In this model, the track modulus ( $u$ ) represents the foundation and comprises the effects caused by sleepers, ballast, sub-ballast, and subgrade (SELIG; WATERS, 1994). The relationship between the support reaction and the rail deflection is determined as follows (HAY, 1982)

$$p(x) = -uy(x), \quad (1)$$

where  $p(x)$  is the force per unit length, proportional to the deflection  $y(x)$ , related through the track modulus ( $u$ ). From this consideration, the differential equation of the beam supported on an elastic base is determined by (HAY, 1982):

$$EIy^{(4)} + uy = 0, \quad (2)$$

where  $E$  is the rail modulus of elasticity, and  $I$  is the rail moment of inertia.

Considering a concentrated load  $P$  applied, as shown in Figure 2, the deflection at any point can be obtained by

$$y = \left( \frac{P}{(64Iu^3)^{1/4}} \right) e^{-\lambda x} (\cos(\lambda x) + \sin(\lambda x)) \quad (3)$$

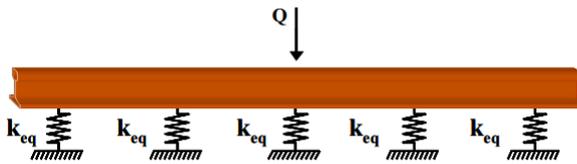
where  $P$  is the wheel load,  $e$  the Euler number,  $x$  is the distance from the load application point to the point where deflection is to be determined, and  $\lambda$  is the damping coefficient, defined by equation (4)

$$\lambda = \left( \frac{u}{4EI} \right)^{1/4}. \quad (4)$$

According to Hay (1982), from equation (3), it is possible to obtain the deflection line of the rail for a given load. If more load is applied, as in the case of the railway axle, a composite deflection line of the rail is determined by superposition, i.e. the ordinates of the lines of each wheel load are added. The formulation presented by Talbot is considered satisfactory for project design situations, and it provides results close to those observed in field tests (HAY, 1982).

In addition to Winkler and Talbot's models, other models have already been developed to analyze load transfer on the railway track superstructure, such as the discrete elastic support model, which considers the track as supported by a series of elastic springs (HAY, 1982). In such case, there is another track stiffness parameter: the support stiffness coefficient ( $k_{eq}$ ) (TEIXEIRA, 2004). Figure 3 represents this model.

**Figure 3** – Rail on discrete elastic supports



**Source:** Adapted from Teixeira (2004), with authorization to reproduce the image.

Teixeira (2004) states that the hypotheses of load distribution under the sleeper, considered in the formulation of the Winkler coefficient, have not yet been verified; therefore, using the support stiffness coefficient may be more adequate as it is a one-dimensional parameter that does not consider the way stress is distributed among the sleepers.

According to Teixeira (2004), Hutter presents a definition of the support stiffness coefficient ( $k_{eq}$ ) considering the vertical stiffness of the track elements through equivalent springs (HUTTER, 1955; TEIXEIRA, 2004), calculated by equation (5):

$$\frac{1}{k_{eq}} = \frac{1}{k_b} + \frac{1}{k_p} + \frac{1}{k_{pa}} + \frac{1}{k_d}. \quad (5)$$

The other parameters, in equation (5), consider the vertical stiffness of the ballast, subgrade, rail fixation, and sleeper and are respectively referred to as ( $k_b$ ), ( $k_p$ ), ( $k_{pa}$ ) and ( $k_d$ ). All of these parameters are encompassed by the spring coefficients with dimension defined by [F] [L]<sup>-1</sup>.

The support stiffness coefficient can also be related to the track modulus ( $u$ ) (TEIXEIRA, 2004):

$$K_{eq} = au, \quad (6)$$

where  $a$  is the spacing between sleepers.

## Methodology

The proposed simulation of the mechanical behavior of the track was carried out by a railway FEM model developed in the Ansys® software, based on the support stiffness coefficient. A three-dimensional model of the rail was built, and the other elements of the superstructure were considered as springs in the sleeper positions.

The results obtained were compared to railway simulation data from related literature. The input data were taken from Silva Filho (2013), who simulated the mechanical behavior of the Carajás Railway, subject to static loading of railway vehicles. The author considered the characteristics of loading, rails, sleepers, ballast, sub-ballast, and subgrade of the track, through a simulation in the Ferrovias 3.0 software, and obtained results of displacements and stresses for the elements of the track.

According to Spada (2003), the Ferrovias 3.0 software calculates the track's response to a given load through a three-dimensional modeling in which the rails and sleepers are modeled using FEM, and the ballast, sub-ballast, and subgrade layers are considered using the finite layer method, incorporating their nonlinear behavior, in a model with 11 sleepers.

The main data presented by the author are described in Table 1.

**Table 1** – Simulation input data

Characteristics	Values
Sleepers spacing	0.61 m
Modulus of rail elasticity	206 GPa
Rail Moment of Inertia	$3.95 \times 10^{-5} \text{ m}^4$
Rail Base Width	0.15 m

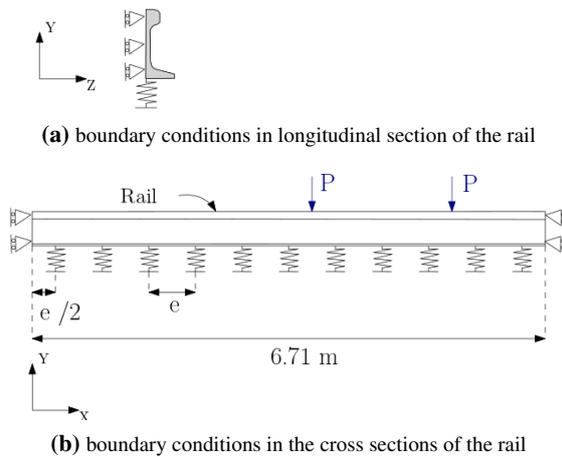
**Source:** Adapted from Silva Filho (2013).

The author presented values for the Poisson's ratio, cohesion and friction angle for the ballast, sub-ballast, and subgrade. It should be noted that these data were not used in this study, since these elements were considered through the track modulus, according to the concept of the support stiffness coefficient.

The data in Table 1 indicates that the characteristics of the rail presented by the author correspond to the TR-68 profile. The rail was simulated as a solid, first designed in CAD (Computer-Aided Design) software, and then imported into finite element software in the 'iges' format. Due to the symmetry of the rail and considering the loading on that axis, only half of the cross section of the rail

was simulated, Figure 4(a). For this reason, symmetry conditions were adopted to prevent displacements in the direction perpendicular to the rail symmetry planes, which are the cross sections, ends of the model, Figure 4(b), and the longitudinal section of the rail, as shown in Figure 4(a). Boundary conditions applied in longitudinal section of the rail are presented in Figure 4(a). Figure 4(b) present the boundary conditions imposed in the cross sections of the model, together with more details as the loads, and spring supports.

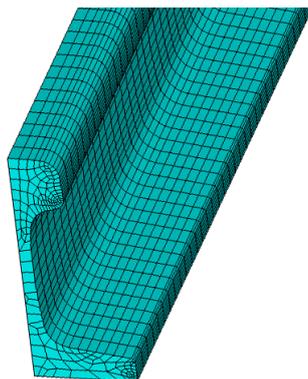
**Figure 4** – Boundary conditions imposed on the rail



Source: The authors.

The Solid 185 element, from the software library, was adopted in the model for being suitable for simulation of solids as it has eight nodes and three degrees of freedom per node (displacements in the  $x$ ,  $y$ , and  $z$  directions). This element allows for plastic, hyperelasticity, creep, large deflection, and large strain analyses. Additionally, in regions of irregular geometry, it is possible to use degenerated forms of the element, such as the prism and tetrahedral forms (ANSYS, 2013). For the rail mesh, 73,920 elements in hexahedral form were used. Figure 5 shows a detail of the rail mesh.

**Figure 5** – Detail of the FEM rail mesh



Source: The authors.

The length of the track assumed for the model was of 6.71 meters, which corresponds to 11 sleepers, as assumed by Silva Filho (2013). The other components of the railway superstructure were assumed as elastic springs, in the sleepers' place. The stiffness of these springs was determined as a function of the sleepers spacing and the track modulus ( $u$ ), as defined in equation (6). Since Silva Filho (2013) did not provide any value for the track modulus, and according to Selig and Waters (1994), it cannot be obtained from the properties of the superstructure components, representative values for the track modulus of the Carajás Railway were researched in the literature.

Many methods to determine the track modulus were developed during the decades (KERR, 2000). According to Costa (2016), this value may vary depending on the load applied on the rail and on the rail conditions. For this railway, he found track modulus ranging from 34 to 84 MPa in stretches that had new ballast. Costa (2016) mentions AREMA's recommendation of 41.4 MPa for track modulus of concrete sleeper railways that are compacted by traffic (AREMA, 2013). As it is not possible to determine one value alone for the railway, the simulation was performed using 41.4 MPa, 60 MPa and 80 MPa for the track modulus, so that a comparative study within the range of values presented by Costa (2016) could be done. Then, their respective support stiffness coefficients are calculated, according to equation (6). They are presented in Table 2.

**Table 2** – Track modulus and its respective support stiffness coefficient

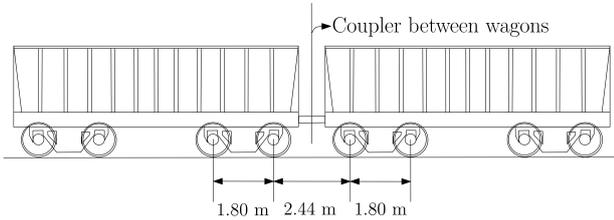
Track modulus ( $u$ ) [MPa]	$K_{eq}$ [MN/m]
41.4	25.2
60	36.6
80	48.8

Source: The authors.

The Combin 14 viscoelastic element, from the software's library, was used to model the springs in the sleepers' place, for it allows to simulate a uniaxial tension-compression spring. This element allows data input of spring stiffness and damping coefficient. In this case, only spring stiffness values were assigned, so that the element could display a uniaxial spring behavior. These elements were created from the nodes located at the base of the rail and nodes 0.5 meters vertically below the rail, which defined the spring support. The spring's height of 0.5 meters was arbitrated, as it does not interfere with the results.

As for the loading, the GDU gondola wagon was considered, the same used by Silva Filho (2013). Figure 6 shows the distance between the wheels at coupling position of the two wagons analyzed.

**Figure 6** – Wheels at coupling position of two GDU wagons



Source: The authors.

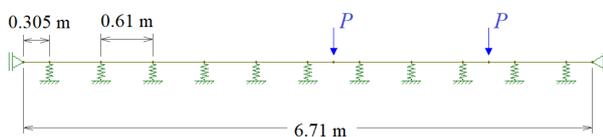
For the GDU wagon, it was assumed the load of 150 tons and wheel load of 196,133 N. The loads were applied and concentrated on the rail head nodes and were simulated as static loads. Only the load value of one wagon was used in the calculation, for the loading system was symmetrical. This way, the location of the coupler corresponded to the end of the model.

Symmetry was considered as a boundary condition for both ends of the model, so that a longer track length can be simulated. The idealized model can be seen in Figure 4, where the position of the loads, the spring supports, the sleepers spacing ( $e$ ) and the model's rail length are represented.

To analyze the results obtained by the proposed model, the displacements in the track were calculated using the Talbot analytical method. Thus, the same track modulus data and the same track characteristics were used. The track deflections lines were obtained from equation (3). As two wheel loads were simulated, the deflection line was composed by the track deflection line for each load.

In addition, an analysis was performed in the FTool<sup>®</sup> software, in which the rail was considered as a beam, and sleepers as spring supports. As boundary condition, restriction of horizontal displacements was arranged at the endpoints of the rail, so symmetry would be preserved in a similar way as it was in the FEM model. The model idealized in the FTool<sup>®</sup> software can be seen in Figure 7.

**Figure 7** – Model analyzed in FTool<sup>®</sup>



Source: The authors.

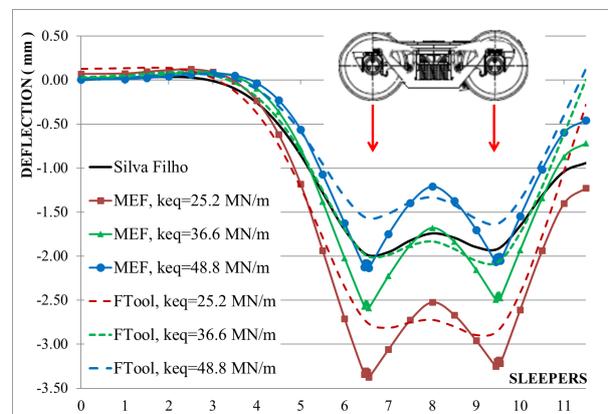
The same characteristics of the rail profile analyzed were used, such as geometry and material properties, as shown in Table 1. Complementary data on the geometry of the TR-68 profile were obtained from NBR 7590 (2012), such as 86.52 cm<sup>2</sup> of cross-sectional area, 18.57 cm of height and 9.84 cm of distance from the gravity center. For the springs, the same stiffness values presented in Table 2 were considered, and the wheel loads were applied in the equivalent positions, to allow the comparison of the displacement results.

## Results

Based on the data and the considerations in the methodology section, the FEM railway model was elaborated, also adding the support stiffness coefficient, in which the effect of a static loading of a GDU wagon was simulated. In this study, the track deflections are analyzed, having as a comparison parameter the results found in the literature and the results of the analytical analysis. In the analysis of the FEM model, it took the computer about 27 seconds (2.4 GHz Core i7 processor, 8 GB RAM and 1.11 GB of data memory) to process the outcome.

First, the track deflection lines for the three values of support stiffness coefficient, presented in Table 2, are obtained. Then, they are compared with the results found by Silva Filho (2013) and those obtained in the FTool<sup>®</sup> model. The results are shown in Figure 8, in which the deflections are plotted as a function of the sleepers, so that the responses delivered in the same points can be compared.

**Figure 8** – Comparison between deflections in the finite element model (FEM) and results found in the literature.



Source: The authors.

According to the results presented in Figure 8, it is noticed that the maximum deflections occur at the point where load is applied. The higher the values of support stiffness coefficient, the lower the deflections, as expected, since they characterize stiffer tracks. Within the three simulated values of support stiffness coefficient, it is noted that for  $k_{eq} = 48.8$  MN/m, the results closest to the reference values, in terms of maximum deflection, were obtained in the FEM model.

In this case, the maximum deflection observed was of 2.13 mm, while in the analysis by Silva Filho (2013) it was 1.98 mm, which corresponds to a difference of 7.6% in the results. Therefore, for  $k_{eq} = 48.8$  MN/m, the maximum deflection at the point of the applied load presented good accuracy. At other points, the displacement was inferior, which shows that the system behavior is stiffer in the proposed model in relation to simulations performed by other researchers.

The greatest differences between the deflections in this analysis and the results achieved in Silva Filho (2013) occur at sleeper number 8, between applications of loads.

At this point, the FEM model proposed presents a deflection 30.5% lower for  $k_{eq} = 48.8$  MN/m. This change in behavior may be due to the characteristics of each model, since Silva Filho (2013) considered the properties and dimensions of the sleepers, ballast, sub-ballast, and subgrade. As for in the proposed model, all these elements are examined in a simpler way, by assuming them as spring's discrete supports, according to the support stiffness coefficients, which were calculated from track modulus values.

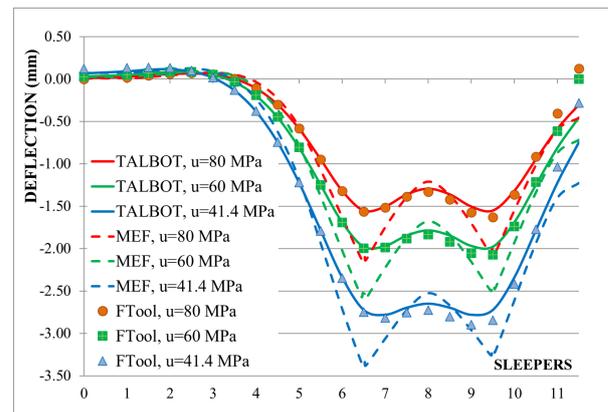
According to Spada (2003), the track modulus represents the characteristics of the sleepers, such as spacing and dimensions, as well as the thickness of the ballast and the stiffness of the subgrade. It assumes all these track characteristics as a single parameter. Employing a single parameter to analyze railway tracks implies a simplified analysis (TEIXEIRA, 2004), which may justify the differences found in the proposed model in relation to the values obtained by researchers in the literature, used as reference.

It is also noticed that the results obtained by the model developed in FTool<sup>®</sup> were closer to the values of reference for  $k_{eq} = 36.6$  MN/m. In general, this model provided deflection lines that were smoother, with values close to the ones of reference, responding less strictly in relation to the 3D FEM model.

The results achieved in this work were also compared with the results obtained in the analytical analysis that

followed the Talbot method. Figure 9 shows the deflections stemmed from the proposed model, as a function of three values of track modulus (Table 2). They are later compared with the deflections of the analytical analysis obtained through equation (3), and with the results delivered by the model developed in FTool<sup>®</sup>.

**Figure 9** – Comparison between deflections in the finite element model (FEM) and in the analytical analysis.



Source: The authors.

From the results presented in Figure 9, it is noticed that the maximum deflections for all three values of track modulus obtained by FEM are greater than those obtained analytically and by the model built in FTool<sup>®</sup>. Thus, the proposed model was more flexible compared to the others analyzed.

It is worth mentioning that hypotheses are different for each model. For instance, in the analytical model, the rail is assumed as continuously supported on an elastic base (Figure 2), while in FEM, the rail is laid on discrete support (Figure 3). Nonetheless, the results obtained by the model developed in FTool<sup>®</sup> were more likely to be akin to the analytical values. The displacement values were slightly greater than the results obtained by the Talbot model. They started to differ greatly in relation to the analytical results from sleeper 10 onwards.

Spada (2003), in a study on the mechanical behavior of a suburban railway network in Rio de Janeiro using field measurements and numerical simulations, concluded that traditional methods, such as the Talbot method, presented results of deflection close to those obtained in the Ferrovias 3.0 software. Thus, the Talbot model and the results provided by the literature used as reference in the present study achieved convergence of solutions and are, therefore, consistent parameters for comparison.

## Conclusion

When comparing the FEM model proposed in this study with the data from literature, it is concluded that the support stiffness coefficient that best matches the simulation by Silva Filho (2013) corresponds to  $k_{eq} = 48.8$  MN/m, for which the maximum deflection is less than 8%. In relation to the deflection line, similar behavior was observed, except for the one found in the position of sleeper number 8, which differed greatly from data presented in previous studies. This divergence may be due to differences between the models. The model proposed here considers only linear elastic behavior for the materials and, in a simple way, it considers the rail as laid on discrete elastic supports. In this model, all the components of the superstructure, but the rail, are evaluated only from the track modulus parameter. Alternatively, the simulation by Silva Filho (2013) considered the elasto-plastic behavior of the ballast, sub-ballast, and subgrade layers, in addition to including the thickness of these layers in the analysis. These disparities contribute to the differences between the models.

The model developed in FTool<sup>®</sup> presented deflection lines significantly similar to the results of Silva Filho (2013) until sleeper number 10, after which the model started to present lower displacements. In this model, the support stiffness coefficient of  $k_{eq} = 36.6$  MN/m resulted in the maximum deflection values closest to the reference.

Compared to the analytical model, the FEM model presented greater deflections for all the track moduli tested, however their deflection lines were similar. Analogous to the comparison with Silva Filho (2013), the deflection results obtained at the position between the loads showed a greater difference in comparison to Talbot model's results. Both models consider the linear elastic behavior of the track; however, the Talbot model considers the rail as continuously supported on an elastic base, while the model simulated in this work considers the rail as laid on discrete elastic support. This divergence between the models may have contributed to the differences found in the deflections.

As expected, the model developed in FTool<sup>®</sup> and the Talbot model had similar displacement responses, since both are simplified models for solving 1D problems (by considering the rail as a beam). Conversely, the FEM model took into consideration the real geometry of the rail; therefore, a 3D analysis was carried out, which may have

contributed to the difference between one-dimensional models (effects from stress concentrations, Poisson's ratio, among others).

Although more advantageous in terms of cost of analysis processing, the models that consider the rail as a beam (Ferrovia and Ftool<sup>®</sup>) are more conservative in their results than the 3D model provided by FEM, in which the simulation of a rail supported by discrete springs is performed. Being more robust, the 3D model will allow future analyses to consider diverse effects on the railway pavement and on the track. Thus, we intend to proceed with the analyses, with the simulation of the rail without the longitudinal symmetry being considered. This way, the loading variable would no longer be simulated at only one node, but in a contact area between the wheel and the rail, taking into consideration the stress distribution in that area, according to the Hertz theory of contact between elastic solids.

Future analyses can also perform three-dimensional simulations of the sleepers and the ballast and sub-ballast layers that consider the interaction between the layers and the properties of each component, with the purpose to obtain an analysis of the mechanical behavior closest to the real one of the whole railway track. The comparison of different models allowed to verify the divergence between their responses and the responses available in the literature, and consequently to calibrate the numerical model.

In general, the model proposed in this study presented consistent results when compared to analytical results and to previous studies, as the deflection lines they provided were similar. Differences were found in the values, which are attributed to the simplifications and considerations adopted in each of the models. Based on the results obtained, it is intended to improve the model in future studies in order to simulate the behavior of a railway track more accurately.

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