

Swarm Interaction in 2D

Interação de Multidão em 2D

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Abstract

In the present work is described a simple minimal model set of ordinary differential system of equations for simulating the swarming behavior of preys under action of predators. Preys and predators are represented by a set of ODEs taking in account the Newtonian attraction-repulsion forces. The predators interacts with the preys through a Newtonian force, which is a nonconservative force (includes friction) that acts in the same direction for both agents. A perturbing force is introduced for the predators' dynamics in order to simulate its behavior among preys. The resulting system of ordinary differential equations is solved numerically by means of Runge-Kutta of fourth order and the dynamics are discussed in the present work as the swarm's ability to realistically avoid the predator. The main goal is to reproduce swarm behavior that has been observed in nature with the minimal and simple possible model of ODE system.

Keywords: Prey-Predator Swarm Interaction. Runge-Kutta 4 Order. Numerical Simulation. Dynamical System.

Resumo

No presente trabalho é descrito um modelo simples mínimo de equações diferenciais ordinárias para simular o comportamento de enxames de presas sob ação de predadores. Presas e predadores são representados por um conjunto de EDOs levando em conta as forças Newtonianas de repulsão-atração. Os predadores interagem com as presas através de uma força Newtoniana, que é uma força não conservativa (inclui fricção) que atua na mesma direção para ambos os agentes. Uma força perturbadora é introduzida para a dinâmica dos predadores, a fim de simular seu comportamento entre as presas. O sistema resultante de equações diferenciais ordinárias é resolvido numericamente por meio de Runge-Kutta de quarta ordem e as dinâmicas são discutidas no presente trabalho como a capacidade do enxame de evitar realisticamente o predador. O objetivo principal é reproduzir o comportamento do enxame que foi observado na natureza com o modelo mínimo e simples possível de sistema EDOs.

Palavras-chave: Interação de Enxames Presa-Predador. Runge-Kutta 4 Ordem. Simulação Numérica. Dinâmica de Sistemas.

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Introduction

In nature it is observed for a long time that animal aggregation is part of the set of individuals behavior (PER-RISH; EDELSTEIN-KESHET, 1999; MOUSSAD et al, 2009; VICSK et al., 1995; RÖNER; TU, 1998). In general is observed fish schooling (shoaling), bird flocking, mammal herding, insect/bacterial swarming, and human crowding dynamics. It is also observed that even predators have been known to hunt in group in the form of packs. It is well known that all these aggregations shares similarities, such as the fact that the group of organisms act in unison and reacting rapidly to obstacles or threats. The generality of such features leads to realize models for its simulation. In the present work, swarming will refer to any such behavior in which individuals come together and act in a reasonably coordinated manner to produce an aggregate set in dynamical motion. Swarming has been studied in an extensive manner by computer simulation (REYNOLDS, 1987; OLFATI-SABER, 2006). In several sources, the models are taken as individual-based, where swarm individuals are represented as a set of individuals that interact with other as a function of their positions (TOPAZ; BERTOZZI, 2005; LETT; MIRABET, 2008). The use of Newtonian force law and variations of it has been a general approach for this dynamical system.

In these models, the designed forces consist of a long-range attractive force that makes the individuals to approach and form the swarm typical geometry, coupled with a short-range repulsive force so that they try do not collide with each other (LIU et al, 2008; DUAN et al, 2005; GAZI; PASSINO, 2004). A self-propulsive force that pushes each individual forward toward some preferred velocity is also often added (D'ORSOGNA et al, 2006), (NISHMURA; IKEGANI, 1997; LEVINE et al, 2000). A model designed to align the individuals with each other is present in flocking simulation (LEE, 2006; INADA; KAWACHI, 2001). All these models successfully reproduced main behavior aspects of swarming. The well known predator behavior called confusion (KRAUSE; RUXTON, 2002), which occurs when the predator is confused related to which individual to pursue is simulated in the present work. Predator confusion acts mainly decreasing its ability to hunt their prey.

Model

A simple but yet robust model is developed in order to approach swarm dynamics. It is designed to represent each prey by a particle that moves based on its interac-

tions with other prey and its interaction with the predator. There is a large material available out there about particle models in biology science, mainly they have been designed to model biological individuals aggregation in general (MOGILNER; EDELSTEIN-KESHET, 1999) also locusts (BERNOFF; TOPAZ, 2011) or fish schooling populations (ZHENG et al, 2005). The model is established as following (CHEN; KOLOKOLNIKOV, 2014). It is assumed that there are N preys whose positions are given by $P(x_j, y_j) \in R^2$, $j = 1, 2, \dots, N$, N is the size of the individual population whereas (x, y) are function of time t . Taking Newton's law so that

$$m \frac{d^2 P_j}{dt^2} + \mu \frac{dP_j}{dt} = F_{j, \text{prey-prey}} + F_{j, \text{prey-predator}},$$

where $F_{j, \text{prey-prey}} + F_{j, \text{prey-predator}}$ is the total force acting on the j -th particle, μ is the strength of friction force and m is its mass. Simplification as the mass m is negligible compared with the friction force μ is applied. After rescaling to set $\mu = 1$ the model is then simplified as

$$\frac{dP_j}{dt} = F_{j, \text{prey-prey}} + F_{j, \text{prey-predator}}.$$

This reduces the second-order ordinary differential system model to a first-order model system, so that the prey moves in the direction of the total force. The prey-prey interaction is given following the form

$$F_{j, \text{prey-prey}} = \frac{1}{N} \sum_{k=1, k \neq j}^N \left(\frac{1}{|P_j - P_k|^2} - a \right) (P_j - P_k).$$

The term $\frac{P_j - P_k}{|P_j - P_k|^2}$ represents Newtonian-type of short-range repulsion that acts in the direction from P_j to P_k , whereas $a(P_j - P_k)$ is a linear long-range attraction in the same direction. The model for prey-predator interactions can be established by a similar manner. In order to deal with more realistic model assume that there is a single predator, however is possible also consider two and/or three predators in the present model. The predator position is given as $PZ_i(x, y, t)$, with $i = 1, 2, 3$. It is considered that the predator acts on the individual's preys as a repulsive particle, it is taken as

$$F_{j, \text{prey-predator}} = b \frac{(P_j - PZ_i)}{|P_j - PZ_i|^2}$$

with b being the strength of the repulsion. Following, the model for the predator-prey interactions as an attractive force given in a similar way such as,

$$\frac{dZ_i}{dt} = F_{j, \text{prey-prey}}.$$

In this case is considered a very simple scenario which $F_{j, \text{prey-prey}}$ is the average over all predator-prey interactions and each individual interaction is a power law, which

decays at large distances, as consequence the prey moves in the direction of the average force. Once these assumptions are put together the following system of equations can be written:

$$\begin{cases} \frac{dP_j}{dt} = \frac{1}{N} \sum_{k=1, k \neq j}^N \left(\frac{(P_j - P_k)}{|P_j - P_k|^2} - a(P_j - P_k) \right) - b \frac{P_j - P_{z_i}}{|P_j - P_{z_i}|^2} \\ \frac{dP_{z_i}}{dt} = \frac{c}{N} \sum_{k=1}^N \frac{(P_k - P_{z_i})}{|P_k - P_{z_i}|^p} \end{cases} \quad (1)$$

As stated before, a is the linear long-range attraction parameter, b is the predator repulsive parameter and c is the predator-prey attraction control parameter, all are positive constants. The system of ordinary differential equations given by equation (1) is then solved numerically by means of Runge-Kutta of fourth order, and it is need to know the t_{ini} for initial time, the end simulation time t_{end} and the number of steps m . This model is also modified making use of a perturbation function added to the predator in order to simulate its decision as the dynamical system evolve in time. The equation (2) are proposed by the author. It is chosen two perturbation functions which are given by:

$$\begin{cases} a) F_{j,pert} = \varepsilon e^{\lambda t} \\ b) F_{j,pert} = \varepsilon (\cos(P_j), \sin(P_j)) \end{cases} \quad (2)$$

In equation (2a) and equation (2b) $\varepsilon = (0.3; 0.2)$, $\lambda = 0.26$. Such constants are arbitrary chosen and fitting each desired simulation.

The present model also implements the density function and area for the swarm population.

Density

It is presented three models for density and the main ideas are from (CHEN; KOLOKOLNIKOV, 2014; PELUPESSY et al., 2013). Here the Kernel functions as described in (GARCIA MARQUEZ, 2014) with modification proposed by the author which is added in present work as follow:

$$\begin{cases} M.\#1 \rho^1 = \frac{1}{2N} \sum_{k=1, k \neq j}^N \lambda_1 e^{\gamma_1 |x_j - x_k|^2} \\ M.\#2 \rho^2 = \frac{(A-B)}{\pi(R_2^2 - R_1^2)} \begin{cases} A = \sum_{k=1, k \neq j}^N e^{|x_j - x_k|^2} i f A < R_1^2 \\ B = \sum_{k=1, k \neq j}^N e^{|x_j - x_k|^2} i f B < R_2^2 \end{cases} \\ M.\#3 \rho^3 = \frac{1}{N} \sum_{k=1, k \neq j}^N \lambda_3 |x_j - x_k| \end{cases} \quad (3)$$

With $\lambda_1 = 1.76$, $\gamma_1 = 1.0$, $\lambda_3 = 15.12$, R_1 and R_2 are described below.

Area

The predator in confused situation (KRAUSE; RUXTON, 2002) generates a ring in a steady state given by the

system equation (1), as shown in the figure 4. Following the definition given in (CHEN; KOLOKOLNIKOV, 2014):

$$R_1 = \sqrt{\frac{b}{a}}, \quad R_2 = \sqrt{\frac{1+b}{a}}$$

where R_1 and R_2 are the internal and external radius defining the steady state annulus for $z = 0$. The steady state occurs when the predator is at the centre of the swarm, surrounded by the prey particles. In such a situation the predator is trapped at the centre of the prey swarm while the prey forms a concentric annulus where the repulsion exerted by the predator cancels out leading to the symmetry population distribution. So, the analytical area is then calculated and compared with the area model given as:

$$Area = \frac{Fs}{N} \sum_{k=1, k \neq j} e^{-\gamma |x_j - x_k|^2} \quad (4)$$

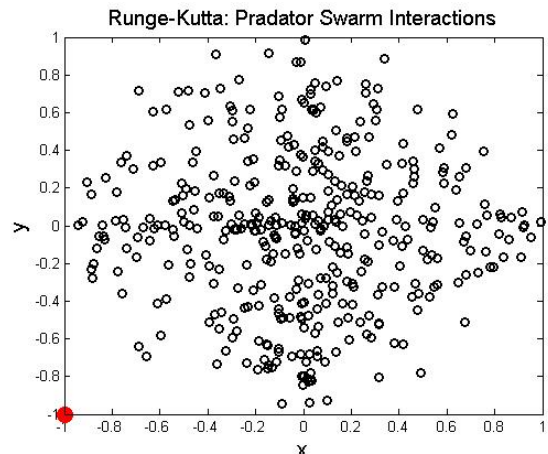
Where F_s is scale factor in order to be able to carry on the comparison with the analytical annulus area and $\gamma = 0.12$.

Many theoretical and computational papers use a routine procedure based on a well-documented method. In such cases, it is sufficient to name the particular variant (referring to key papers in which the method has been developed), to cite the computer program used, and to indicate briefly any modifications made by the author.

Numerical Simulation

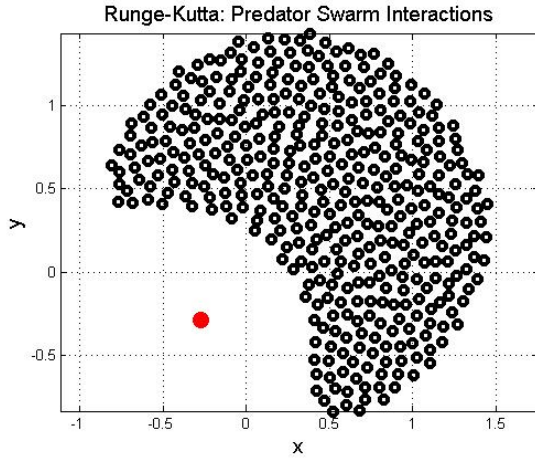
The numerical simulation is performed taking the following parameters as constant: $p = 2.4$, $a = 1$, $b = 0.5$, $t_{ini} = 0.0$, $t_{end} = 12.0$, $N = 400$ (particles), $m = 480$ (time steps). The first simulation is without perturbation function. The initial position conditions are generated by random number scaled between 0 and 1, as shown in figures 1, 5 and 9.

Figure 1: $c=2.8$, $t=0.0s$.



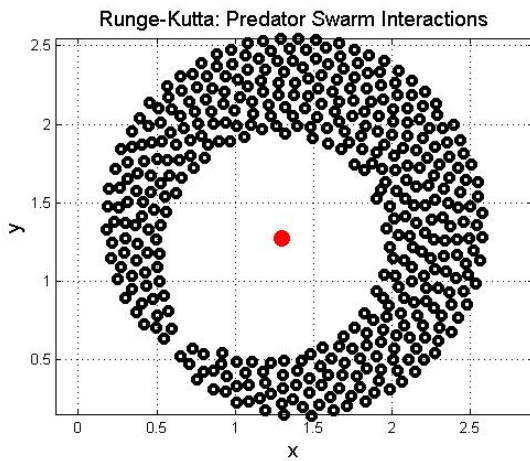
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Figure 2: $c=2.8, t=2.0s$.



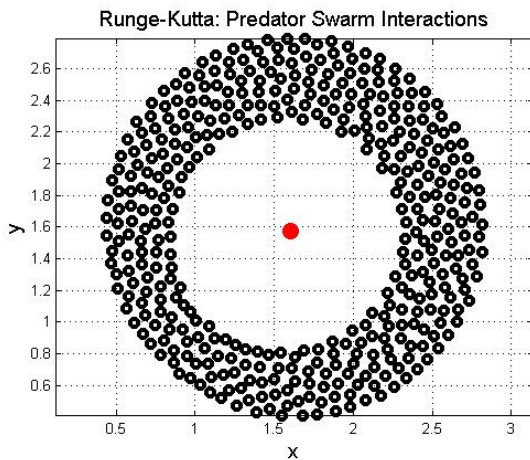
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Figure 3: $c=0.8, t=8.0s$.



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Figure 4: $c=0.8, t=12.0s$.



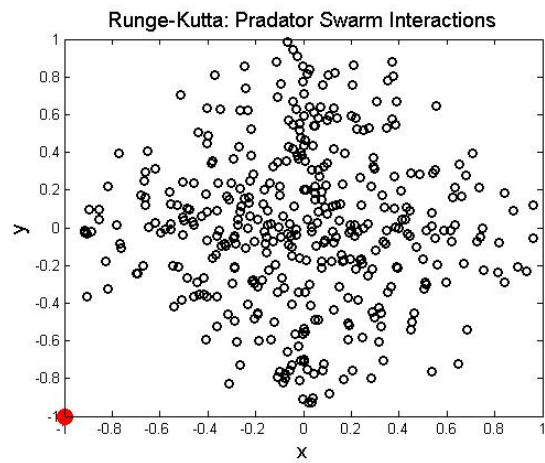
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In the figure 1 is depicted the initial distribution of the 400 individuals (black dots). Figure 2 shows an elapsed time of 2.0s showing the natural repulsive presence of (the

red dot) the predator and the protective action of the preys as the predator moves towards the population. In figure 3 the predator is surrounded by the prey and its movement slow down leading to a confused situation as depicted in figure 4 after 12.0s in these both case $c = 0.8$. In this stage the system becomes stable and the predator is kept in confused position (CHEN; KOLOKOLNIKOV, 2014).

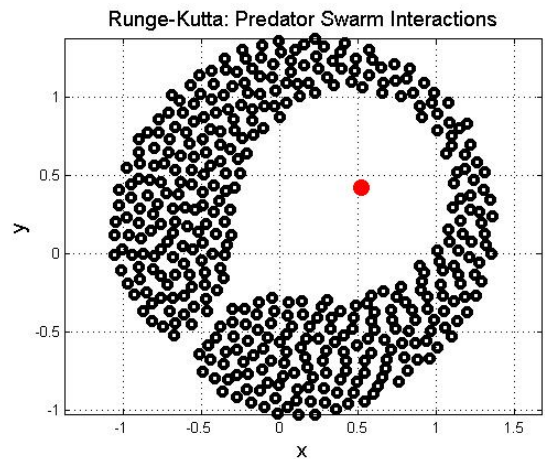
The following numerical simulation is performed using the perturbation function given by equation (2a). It is considered the model parameters as constant: $p = 2.4, a = 1, b = 0.5, t_{ini} = 0.0, t_{end} = 6.0, N = 400$ (particles), $m = 480$ (time steps).

Figure 5: $c=2.8, t=0.0s$.



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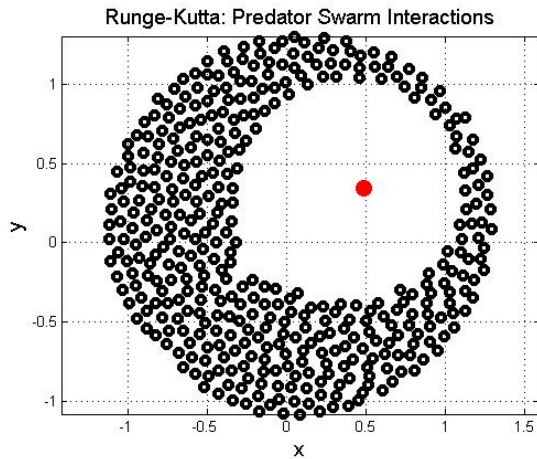
Figure 6: $c=2.8, t=2.0s$.



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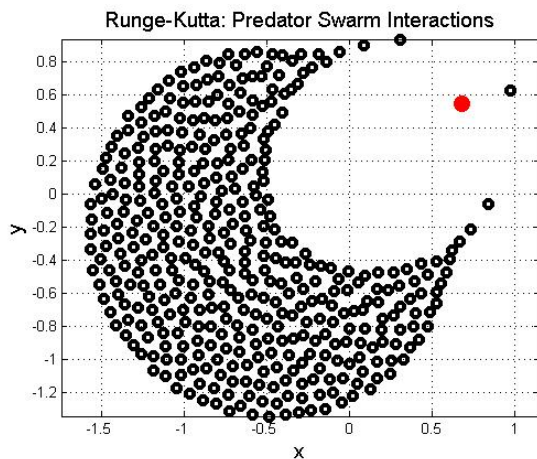
In the figure 5 is depicted the initial distribution of the 400 individuals. Figure 6 shows an elapsed time of 2.0s showing the invasive movement of the predator towards the preys its natural repulsive presence of (the red dot) as the predator and the protective action of the preys forming a circle leading the predator to a completed surrounded

Figure 7: $c=2.8, t=3.0s$.



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Figure 8: $c=2.8, t=6.0s$.



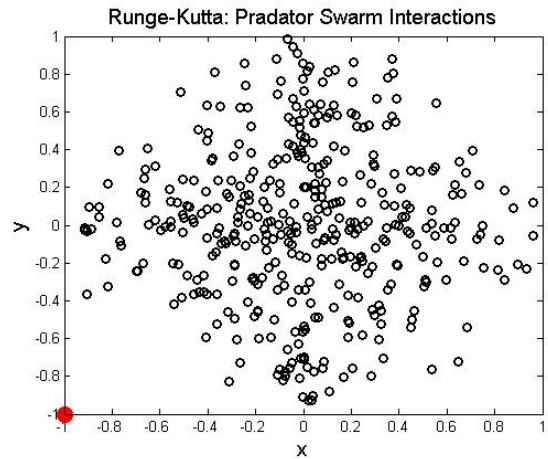
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situation. In figure 7 the predator is already surrounded by the preys and its attack movement is kept leading to a final chasing as depicted in figure 8 after 6.0s ending up to catch up. In this stage the system as propagated in time tends to regroup the prey population.

The numerical simulation using the perturbation function given by equation (2b) is performed taking the parameters as used for equation (2a).

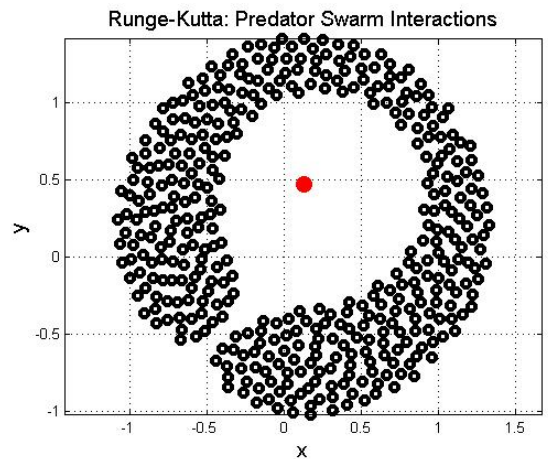
In the figure 9 is depicted the initial distribution of the 400 individuals. Figure 10 shows an elapsed time of 2.0s showing the invasive movement of the predator towards the preys population. In this case the perturbation function simulates a more aggressive behavior of the predator leading it to an oscillating movement approaching the preys faster than for the perturbation function given by equation (2a). Dynamically in real time processing is visible the preys evasive actions as time evolves. In figure 11 the predator is already surrounded by the preys and its

Figure 9: $c=2.8, t=0.0s$.



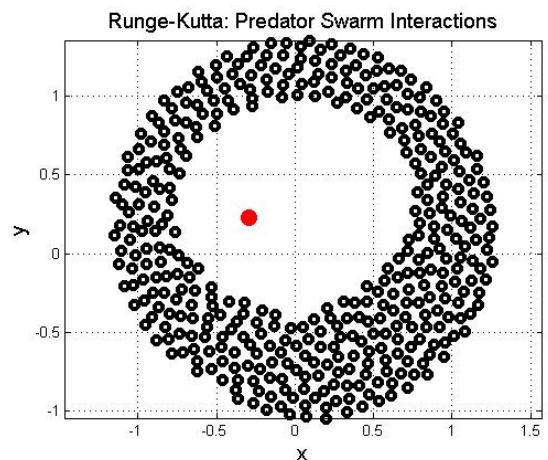
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Figure 10: $c=2.8, t=2.0s$.



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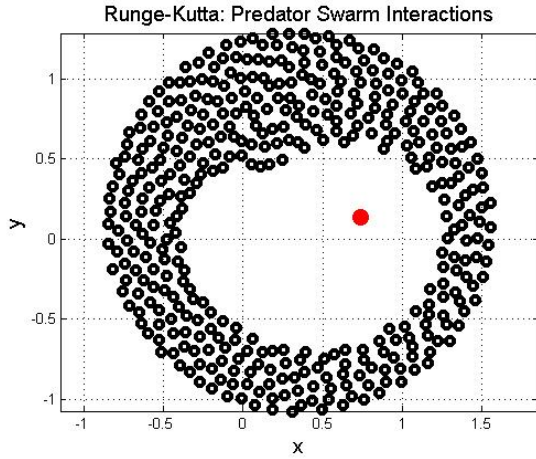
Figure 11: $c=2.8, t=3.0s$.



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attack movement is continuous as depicted in figure 12 after 6.0s where the preys system becomes more unstable due more aggressive predator behavior. Qualitatively the

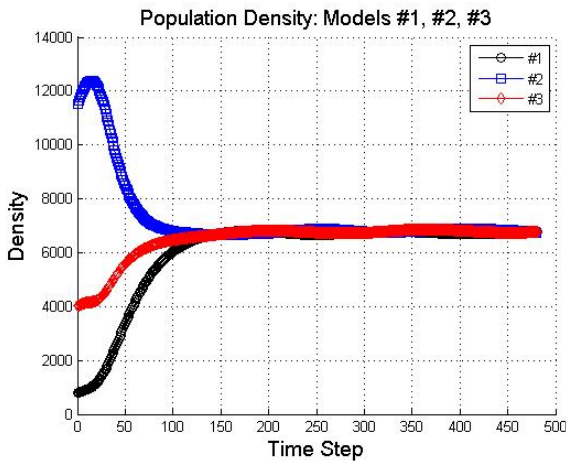
Figure 12: $c=2.8, t=6.0s$.



Source The author.

results shown here is in agreement with those of presented by (CHEN; KOLOKOLNIKOV, 2014). In the figure 13 and figure 14 is shown the density and area calculation, for the case with perturbation function, extending the final time to 8.0s.

Figure 13: $c=2.8, t=8.0s$.



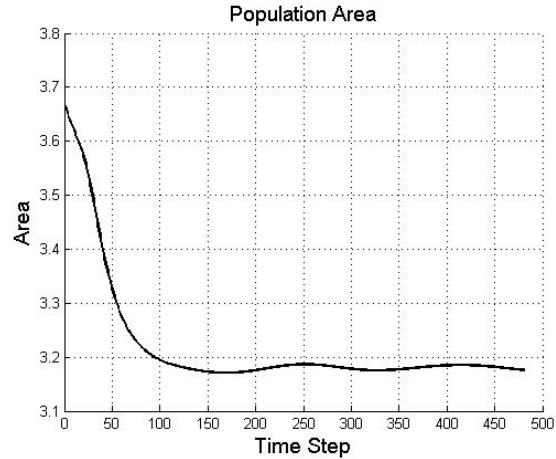
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As defined by equation (3): $M.\#1$ Density = 6781.499598. $M.\#2$ Density = 6753.253767. $M.\#3$ Density = 6770.577087. Analytical Total Area = 3.141593. Numerical Total Area = 3.175787.

In the figures 15 and 16 are depicted the external and internal boundaries making use of the Alpha Shape curve algorithm as described by (EDELSBRUNNER, 1995).

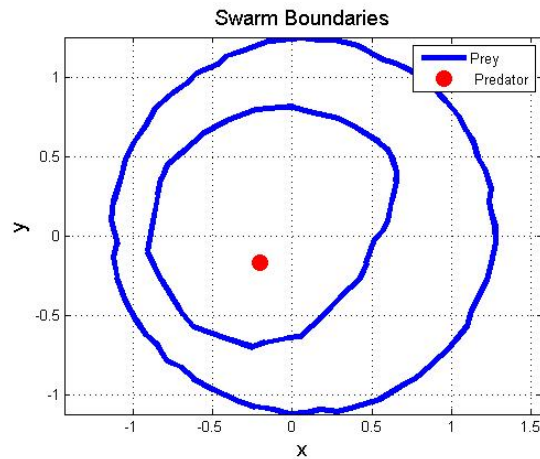
The last two results depicted in figure 17 and 18 show the situation where there are three predators acting on the population. The dynamics of this case is quite interesting because the predator action leads the preys to take fast evasive movements producing intricate swarm patterns. The same perturbation function is also used for all three

Figure 14: $c=2.8, t=8.0s$.



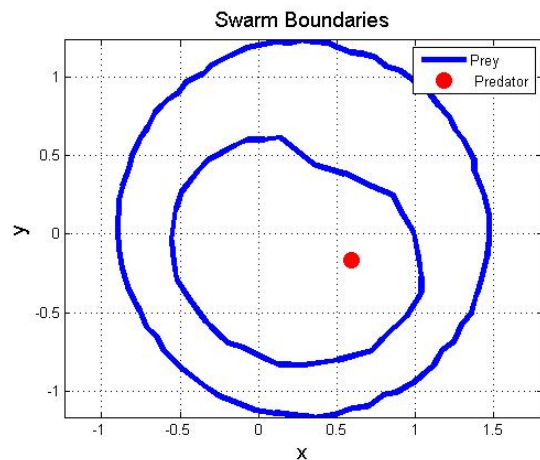
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Figure 15: $c=2.8, t=6.0s$.



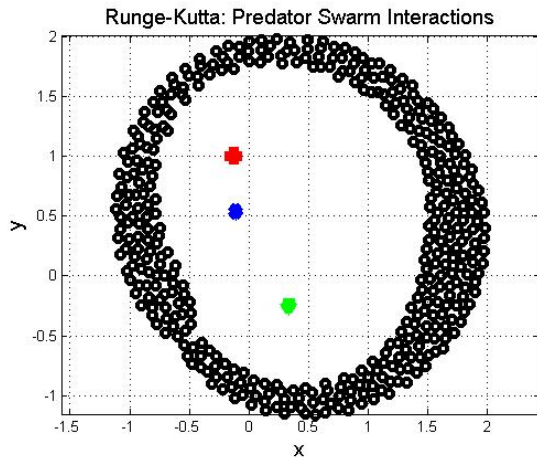
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Figure 16: $c=2.8, t=8.0s$.

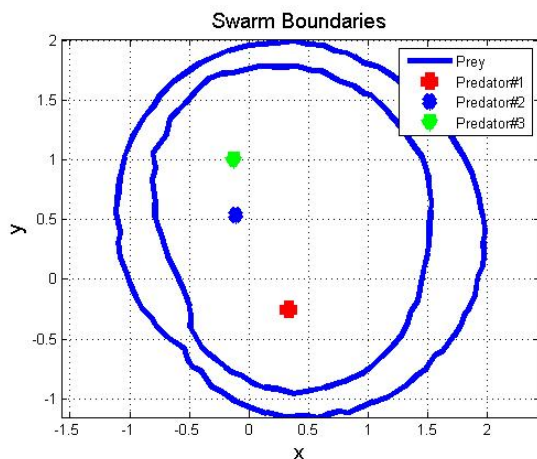


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predators as before. The time shown is 4.0s for this swarm pattern.

Figure 17: 3 predator and swarm.

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Figure 18: Boundaries.

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Conclusion

The numerical simulation is well done even for this large system of ordinary differential equations (806 equations), the RK-4 is robust enough to deal with it, since the time step used is small enough to follow the predator-prey engagement dynamics. One must care for this fact because the predator dynamics is more sensitive to the time step used as for preys' dynamics. The present model is completely able to predict the swarm dynamics and through these simulations became clear that the b and c parameter has deep influence in this dynamics system. Lower c values (< 2.8) also acts in lower down the predator movement towards the prey population. The adoption of the perturbation function for the predator also revealed that the function given by equation (2b) makes the predator to move towards the preys in a irregular path. This causes the prey population take a fast evasive movement always

keeping the safe population shape of protection which tray to involve the predator in a confused position. But this perturbation function leads the prey to change rapidly in evasive movements. However, the perturbation function given by equation (2a) implies a smooth path movement for the predator leading to a stable prey response. So, the equation (2b) seems to be more realistic even than for three predators. The density models converge to a stable value as the area calculation is in good agreement with the prescribed analytical value.

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Received: Sept. 11, 2018
Accepted: Dec. 12, 2018