Vapour and air bubble collapse analysis in viscous compressible water

Análise do colapso de uma bolha de vapor e ar em água viscosa e compressível

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Abstract: Numerical simulations of the collapse of bubbles (or cavities) are shown, using the finite difference method, taking into account the compressibility of the liquid, expected to occur in the final stages of the collapse process. Results are compared with experimental and theoretical data for incompressible liquids, to see the influence of the compressibility of the water in the bubble collapse. Pressure fields values are calculated in an area of 800 x 800 mm, for the case of one bubble under the hypothesis of spherical symmetry. Results are shown as radius versus time curves for the collapse (to compare collapse times), and pressure curves in the plane, for pressure fields. Such calculations are new because of their general point of view, since the existing works do not take into account the existence of vapour in the bubble, neither show the pressure fields seen here. It is also expected to see the influence of the compressibility of the water in the collapse time, and in the pressure field, when comparing pressure values.
Key words:

Resumo: São apresentadas simulações numéricas do colapso de bolhas, ou cavidades, utilizando o método das diferenças finitas, levando em consideração a compressibilidade do líquido, o que se espera que ocorra nos estágios finais do processo de colapso. Os resultados obtidos são comparados com dados experimentais existentes, e com dados teóricos para líquidos incompressíveis, visando verificar a influência da compressibilidade da água no colapso da bolha. O campo de pressões é calculado em uma área de 800 x 800 mm, para uma bolha, com a hipótese de simetria esférica. Os resultados são apresentados na forma de gráficos raio versus tempo para o colapso (para se comparar os tempos de colapso), e isobáricas para o campo de pressões. As novidades neste trabalho devem-se ao fato de que os cálculos existentes não costumam levar em consideração o vapor dentro da cavidade, nem apresentam isobáricas na forma aqui mostrada. Espera-se verificar a influência da compressibilidade da água sobre o tempo de colapso, e sobre o campo de pressões.
Palavras-chave:

1 Introduction

In the process of formation of bubbles in liquids, air and vapour are always trapped inside the bubbles, since the nucleation begins in a micro-bubble of air (HAMMITT, 1980), and the bubble is filled by vapour as it grows. So the presence of vapour must be taken into account as well as air. Poritsky (1952), when studying the collapse of a bubble, considered a pressure inside the bubble, but did not take into account the compression of vapour and air during the collapse.

When the bubble is submitted to higher pressures, collapse will occur. Some photographic studies of collapsing bubbles on a flow over ogives were made by Plesset (1949), and nozzle flows calculations under cavitating conditions were made by Delale *et al.* (2001). But in the present work, no body geometry is considered, just the collapse itself and the resulting pressure field. The pressure field due to the collapse of bubbles in liquids can be calculated as a function of the radius of the bubble and the physical properties of the fluid, for special values of initial and some boundary conditions. Some of these pressure field calculations can be seen in Bazanini *et al.* (1999), for incompressible liquids.

Calculations considering the compressibility of the liquid and some of the physical properties of the fluid have been done in the last decades, beginning with Trilling (1952), and Keller; Kollodner (1956). The inclusion of the Mach number in the Rayleigh-Plesset equation was made by Hickling ; Plesset (1964), and with numerical simulations by Ivany; Hammitt (1965). A good review of the subject appears in Neppiras (1980), showing oscillations of stable cavities. Further discussions of bubbles in sound fields can be seen in Prosperetti (1984), and Young (1989), but none of the works take into account the compression of the vapour, considered in the present work. Recent studies involving the sonoluminescence phenomenon (conversion of sound into flashes of light in the liquid, caused by bubble oscillations, and not completely explained by original equations of fluid mechanics) such as those

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by Barber *et al.* (1997), and Dan *et al.* (2000), have been creating a raising interest on cavitation research again, since it is expected the phenomena are correlated.

Pressure field shall be calculated for water, under the hypothesis of adiabatic collapse, since there is no time for heat transfer to occur. Better results for collapse for adiabatic hypothesis when compared to isothermal hypothesis, was already made by Bazanini *et al.*, (1998), and Bazanini *et al.* (1999). The purpose is to compare the results obtained here with those ones, to see the influence of the compressibility of the water on the pressure field, and also on the collapse time of the bubble.

2. Basic Equations

2.1 Collapse equation

The equation for the motion of the bubble wall during the collapse of a spherical bubble in a compressible liquid can be seen in Löfstedt *et al.* (1993), in the form below:

$$\left(1-\frac{2R'}{C}\right)RR''+\frac{3}{2}\left(1-\frac{4R'}{3C}\right)R'^2=\frac{1}{\rho_L}\left[P_L+\frac{R}{C}\left(1-\frac{R'}{C}\right)\frac{dP_L}{dt}-P_\infty\right]. \tag{1}$$

where C is the sound velocity, R is the bubble radius, R' is its time derivative, ρ_L is the liquid density, and P_L and P_∞ are the liquid pressure close to the bubble wall and the pressure in a position far enough in the liquid (where no effects of the collapse are felt), respectively.

The acoustic approximation (constant sound velocity), as described by Neppiras (1980), shall be used here, but surface tension and viscosity will not be neglected.

Considering the physical properties of the fluid, and the presence of vapour and air in the bubble, the liquid pressure is:

$$P_L=\frac{P_{g_0}R_0^{\,3K_g}}{\left(R^3-a_g^3\right)^{K_g}}+\frac{P_{v_0}R_0^{\,3K_v}}{\left(R^3-a_v^3\right)^{K_v}}-\frac{2S}{R}-\frac{4\left(\mu_g+\mu_L\right)}{R}R' \tag{2}$$

where m_g and m_L are the gas (air) and liquid viscosity, respectively. P is the pressure and S is the surface tension. The hard core radius for the air, a_g, is considered here, as described by Barber *et al.* (1997), and Yasui (1998). Since the presence of vapour is made here, a_v must be considered too. Air and vapour initial pressure inside the bubble, P_{g0} and P_{v0}, respectively, shall be considered as well as air and vapour adiabatic constants, K_g and K_v.

Thus for the derivative term of the pressure liquid, considering all physical properties involved in the process, results:

$$\frac{dP_L}{dt}=-3R^2R'\left[\frac{K_gP_{g_0}R_0^{3K_g}}{\left(R^3-a_g^3\right)^{1+K_g}}+\frac{K_vP_{v_0}R_0^{3K_v}}{\left(R^3-a_v^3\right)^{1+K_v}}\right]+\frac{2SR'}{R^2}-4(\mu_g+\mu_L)\left(\frac{RR''-R'^2}{R^2}\right) \tag{3}$$

2.2 Pressure field equations

Available method for pressure field calculation during the collapse of a bubble is appropriate for one empty bubble only, and disregards physical properties of the fluid, as can be seen in the classical work by Rayleigh (1917).

To calculate the pressure field taking into account the physical properties of the fluid, bubbles shall be assumed as sinks in the potential flow theory. The pressure field will be calculated in an area of 800 x 800 mm, using a mesh size of 1 x 1 mm. For the two-dimensional case, Navier-Stokes and continuity equations are (WELTY *et al.*, 1984):

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_L}\frac{dP}{dx}+\nu_L\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}\right) \tag{4}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho_L}\frac{dP}{dy}+\nu_L\left(\frac{\partial^2v}{\partial x^2}+\frac{\partial^2v}{\partial y^2}\right) \tag{5}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

where u and v are the velocities in the x and y direction, respectively, and n_L is the kinematic viscosity of the liquid.

Differentiating equation (4) with respect to x and equation (5) with respect to y, adding both resulting equations we set

$$u\frac{\partial^2u}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+v\frac{\partial^2u}{\partial y\partial x}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+u\frac{\partial^2v}{\partial x\partial y}+v\frac{\partial^2v}{\partial y^2}=-\frac{1}{\rho_L}\left(\frac{\partial^2P}{\partial x^2}+\frac{\partial^2P}{\partial y^2}\right)+ \tag{7}$$

$$v\left[\frac{\partial}{\partial x}\left(\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}\right)+\frac{\partial}{\partial y}\left(\frac{\partial^2v}{\partial x^2}+\frac{\partial^2v}{\partial y^2}\right)\right]$$

Rearranging the terms results the following equation

(8)

$$\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+u\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+$$

$$v\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}=$$

$$-\frac{1}{\rho_L}\left(\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}\right)+\nu\left[\frac{\partial}{\partial x^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial y^2}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]$$

Now simplifying using equation (6), we obtain equation (9) below

(9)

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}=-\frac{1}{\rho_L}\left(\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}\right)$$

Once again rearranging the terms:

(10)

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=-2\rho_L\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}-\rho_L\left(\frac{\partial u}{\partial x}\right)^2-\rho_L\left(\frac{\partial v}{\partial y}\right)^2$$

The flow function for the two-dimensional case, as defined by Swansom (1970), is

$$u=\frac{\partial \psi}{\partial y} \quad (11)$$

$$v=-\frac{\partial \psi}{\partial x} \quad (12)$$

Using the flow function definition above, equation (10) becomes:

(13)

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}=2\rho_L\left[\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial y^2}-\left(\frac{\partial^2 \psi}{\partial x\,\partial y}\right)^2\right]$$

The use of this equation can be made for any number of bubbles, as seen in Bazanini *et al.* (1999). Once the bubbles are treated as sinks, the flow function field can be obtained simply adding the flow function for every bubble. Flow function field calculations for every sink can be made using the following equation

$$\psi=-RR'C \quad (14)$$

where *C* is the position of the calculated point related to the sink.

3 Results

To obtain radius versus time curves, equation (1) was solved using the finite difference method in an explicit time integration scheme, for a step of 10^{-5} s, accurate for the process of collapse, and initial conditions described in the following paragraph. For comparison, was used the value of 27,579 Pa for P_8, the same used by Knapp ; Hollander (1948). The values of hard core *a* for air and water vapour used here can be seen in Table 1, where the value for air was extracted from Löfstedt *et al.* (1993). Results are compared with experimental data and theoretical calculations for isothermal and adiabatic hypothesis, as shown in Figure 1. Isothermal collapse was calculated by Bistafa; Bazanini (1997), and adiabatic collapse calculations for incompressible water are extracted from Bazanini *et al.* (1998).

To calculate the pressure field it is necessary to solve equations (1) and (13), whose analytical solutions are difficult to find. Therefore let us use numerical methods to solve these equations. Equations (14) and (13) were solved using the finite difference method, for the following conditions: R_0 = 3,56 mm, P_{g0} = 40 Pa (initial conditions), the same used by Knapp; Hollander (1948), for comparison purposes; P¥ = 50,000 Pa (boundary condition); step = 10^{-7} s, enough for such calculations. For the compressible case, it was considered the 'acoustic approximation', taking the sound velocity as 1481 m/s, used by Löfstedt *et al.* (1993). For such conditions, the minimum bubble radius is about 1 mm. Then the bubble begins to growth, since internal pressure values become great enough for that, due to the compression process. This phenomenon is known as rebound, and is discussed in works such as those by Fujikawa ; Akamatsu (1980), Hickling ; Plesset (1964), and Lauterborn; Ohl (1997), but the scope of this work is the main collapse (the first one). Results are shown in Figures 2, 3 and 4, the first one valid for incompressible and compressible liquids, since no difference can be seen yet. Figures 3 and 4 are for incompressible and compressible liquids, respectively. Figure 3 was extracted from Bazanini *et al.* (1999). Although continuity equation appears in the form used for incompressible liquids, the compressibility is considered in equation (1), through the use of the sound velocity *C*. In Table 1 below, values of initial vapour and air pressure are shown. Vapour pressure has great influence in the pressure field, once, as vapour and air are compressed as the bubble collapses, vapour and air pressures inside the bubble raise, and initial vapour pressure is much greater than initial air pressure, as can be seen in Table 1 below.

Table 1 – Initial air and vapour pressure, and hard core a

Fluid	P_0 (Pa)	R_0/a
Water	2,340	10.79
Air	40	8.54

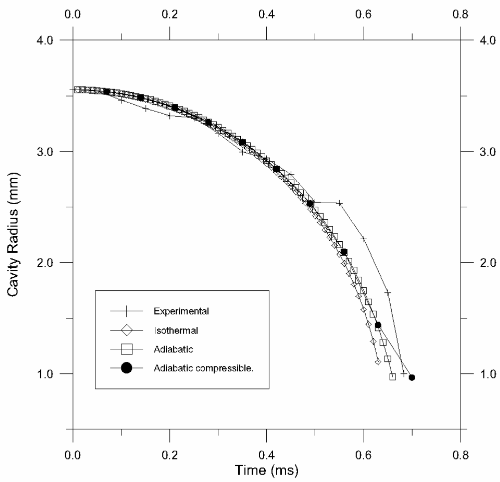

Figure 1 – Comparison between experimental and calculated data.

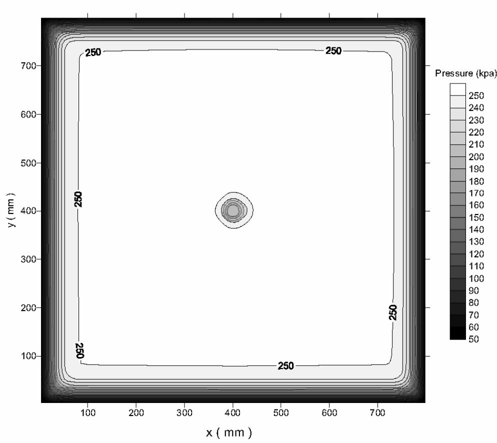

Figure 2 – Pressure field for compressible and incompressible water, 0.58 ms of collapse.

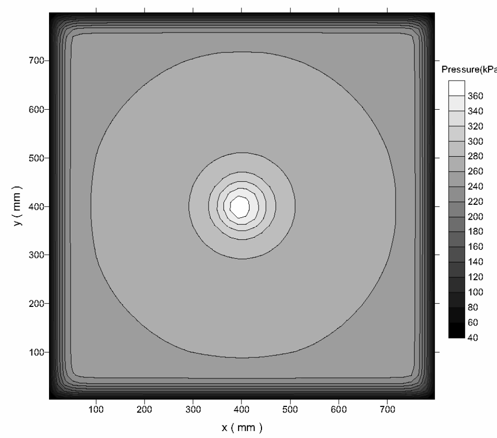

Figure 3 – Pressure field for incompressible water, 0.65 ms of collapse, bubble radius of 1 mm.

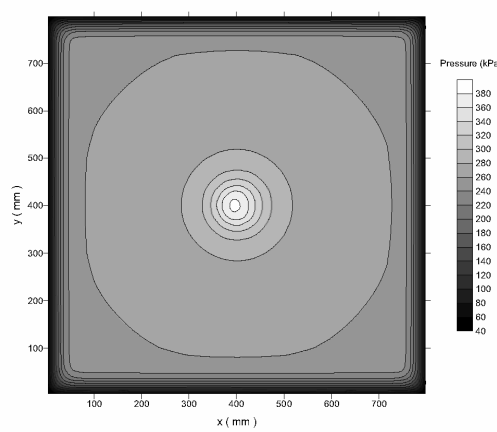

Figure 4 – Pressure field for compressible water, 0.7 ms of collapse, bubble radius of 1 mm.

4 Conclusions

Adiabatic collapse showed a better agreement to experimental data than isothermal collapse because the process is too fast for heat transfer to occur (about 0,7 ms). From Figure 1, one can conclude that compressibility effects appear as the collapse proceeds, specially in the final portion of the collapse, as predicted by Young (1989). This can be seen in Figure 2 (valid for compressible and incompressible liquid), for a collapse time of 0.58 ms, when the bubble radius is approximately 2 mm (see Figure 1), and no compressibility effects are felt, since pressure inside the bubble is still low . All results from calculations showed a good agreement with experimental data, specially the adiabatic collapse in compressible liquid, that is the best approximation obtained by the author until here. The pressure field Figures 3 and 4 are for a radius of 1 mm, which means a collapse time of about 0.7 ms for the compressible case (Figure 4), and about 0.65 ms for the incompressible case (Figure 3). Experimental values correspond to 0.69 ms approximately (from Figure 1), closer to a more complete formulation, i.e., the adiabatic compressible case. Initial vapour pressure, disregarded in the works referenced here, is of major influence in the pressure field of collapsing spherical bubbles. This fact is due to the compressing process during the collapse of the bubble. As the bubble collapses, vapour and air are compressed and pressures inside the bubble and in the liquid raise. It can be clearly seen that greater values of pressure are for smaller bubble radii. We have grater pressures in Figures 3 and 4 (1mm bubble radius), than in Figure 2, which was calculated for a greater value of bubble radius (2 mm).

Comparing Figures 3 and 4, we can see a small increase in the values of the pressure, for the compressible case. Although there is no experimental values of pressure field to compare with results obtained in the form presented here, it was possible to compare calculated values between the incompressible and the compressible case. According to Neppiras (1980), as the collapse proceeds the pressure peak increases in height and moves closer to the bubble wall. The thickness of the high pressure region also decreases. Such phenomena is verified in the present work, when comparing Figures 3 and 4. For the compressible case, since the collapse is longer (Figure 1), it is expected higher values of pressure, confirmed by Figures 3 and 4. Calculations presented here are adequate to any fluid of known physical properties, but there are an infinity number of possibilities for calculations when varying parameters such as number and positions of bubbles, initial and boundary conditions, and physical properties. Such versatility is of great value in the method used here, but there are no experimental pressure fields values in the form presented in Figures 2 to 4. So, although the pressure values calculated here are the expected ones, it is impossible to make more comparisons, except those ones made here.

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