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THE CONSTRUCTION OF GEOMETRIC MENTAL RELATIONSHIPS IN A PATTERN BLOCK PUZZLE ACTIVITY AT AGE FOUR

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Abstract: Using constructivist scholar Constance Kamii's perspective on autonomy, physical, social, and logico-Mathematical knowledge, this research investigates a curriculum method developed to engage young children in the construction of geometric knowledge. To study this method, I designed a seven-week pattern block puzzle intervention with four-year-old children in a constructivist classroom. All children were considered at-risk for school failure. The investigation is rooted in my unsatisfying experiences with pattern blocks in my preschool classrooms. Data were obtained from pre and posttests assessment administered to ascertain effects of the intervention on all children and from observations of children's behavior as they engaged in the pattern block activity. Piaget's (1936/1952), Piaget and Inhelder, 1948/1956) theory of knowledge and intelligence was the framework for detailed qualitative analysis of one exemplar's progress during the intervention. Results of pre and posttests and microanalysis indicate children made significant progress in their construction of geometric knowledge. Children learned to match shapes with corresponding spaces and distinguish among and coordinate the sizes of angles and spaces. I describe using a series of detailed drawings, one exemplar's actions during the activity. I conclude with suggestions for further research and educational implications.

Key-words: constructivist; pattern block puzzles; geometric; microanalysis.

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A CONSTRUÇÃO DE RELAÇÕES MENTAIS GEOMÉTRICAS EM UM PADRÃO DE ATIVIDADE DE QUEBRA-CABEÇAS AOS QUATRO ANOS

Resumo: Utilizando a perspectiva da pesquisadora construtivista Constance Kamii sobre autonomia, conhecimento físico, social e lógico-matemático, esta pesquisa investiga um método curricular desenvolvido para envolver crianças pequenas na construção do conhecimento geométrico. Para estudar esse método, foi projetada uma intervenção de sete semanas com com quebra-cabeça em bloco para crianças de quatro anos, em uma sala de aula construtivista. Todas as crianças foram consideradas como de risco para fracasso escolar. A investigação está enraizada nas minhas experiências insatisfatórias sobre os padrões aplicados nas minhas salas de aula pré-escolares. Os dados foram obtidos a partir da avaliação pré e pós-teste aplicada para verificar os efeitos da intervenção em todas as crianças e a partir de observações do comportamento das crianças que se envolveram na atividade. A teoria do conhecimento e da inteligência de Piaget (1936/1952), Piaget e Inhelder, 1948/1956) foi o marco para a análise qualitativa detalhada do progresso exemplar durante a intervenção. Resultados de pré e pós-testes e microanálises indicam que as crianças fizeram progressos significativos em sua construção do conhecimento geométrico. As crianças aprenderam a combinar formas com espaços correspondentes e distinguir e coordenar os tamanhos dos ângulos e espaços. Descrevo usando uma série de desenhos detalhados, das ações de um exemplar durante a atividade. Concluo com sugestões para novas pesquisas e implicações educacionais.

Palavras-chave: construtivismo; quebra-cabeças de blocos; geometria; microanálise

LA CONSTRUCCIÓN DE RELACIONES MENTALES GEOMÉTRICAS EN UN PATRÓN DE ACTIVIDAD DE ROMPECABEZAS A LOS CUATRO AÑOS

Resumen: Utilizando la perspectiva de la investigadora constructivista Constance Kamii sobre autonomía, conocimiento físico, social y lógico-matemático, esta búsqueda investiga un método curricular desarrollado para involucrar a niños pequeños en la construcción del conocimiento geométrico. Para estudiar este método, se diseñó una intervención de siete semanas con rompecabezas en bloque para niños de cuatro años, en un aula constructivista. Todos los niños fueron considerados como de riesgo para fracaso escolar. La investigación está enraizada en mis experiencias insatisfactorias sobre los estándares aplicados en mis aulas preescolares. Los datos se obtuvieron a partir de la evaluación pre y post-test aplicada para verificar los efectos de la intervención en todos los niños y a partir de observaciones del comportamiento de los niños que se involucraron en la actividad. La teoría del conocimiento y de la inteligencia de Piaget (1936/1952), Piaget e Inhelder, 1948/1956) fue el marco para el análisis cualitativo detallado del progreso ejemplar durante la intervención. Los resultados de pre y post-pruebas y microanálisis indican que los niños han hecho progresos significativos en su construcción del conocimiento geométrico. Los niños han aprendido a combinar formas con espacios correspondientes y a distinguir y coordinar los tamaños de ángulos y espacios. Describo usando una serie de dibujos detallados, de las acciones de un ejemplar durante la actividad. Concluyo con sugerencias para nuevas investigaciones e implicaciones educativas.

Palabras-clave: constructivismo; rompecabezas de bloques; geometría; microanálisis

Introduction

"Teacher, will you help me?" "Sure," I say and move to sit closer to Noah. I observe his actions. Now and then I make a suggestion. Each is either ignored or met with a resounding, "Noooooo!" Because this interaction occurred frequently in the first seven sessions I worked with him, I asked myself, "Why does Noah ask for help and then repeatedly refuse to accept it? How can I help this child when he resists every intervention I try? What is going on in his mind? Is he learning anything? If so, what is he learning?" Although the first two questions were new, I had asked the last three many times before. Those questions were the reason I was conducting research using pattern blocks and pattern block puzzles.

Pattern blocks were developed by engineers and educators in the 1960s at the Education Development Center in Waltham, Massachusetts. They are widely found in early childhood classrooms. Since the early years of my teaching, I have had the pattern blocks in my classroom (see Figure 1).

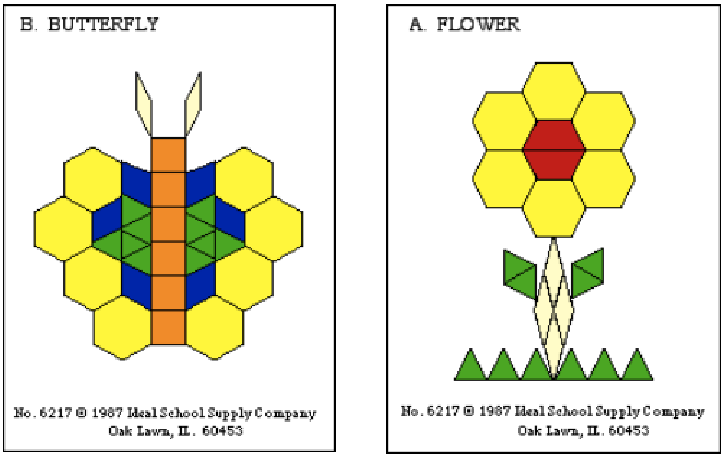
Figure 1 - Shapes in a pattern block set

Source: research collection

A set of pattern blocks consist of 250 brightly colored shapes: 25 yellow hexagons, 50 read trapezoids, 50 green triangles, 50 blue rhombuses, 50 tan rhombuses, and 25 orange squares. All shapes have one-inch sides except for the trapezoid. The trapezoid has three sides measuring one inch and one side measuring two inches. I hoped children would learn something from playing with them, although at the time, I was not sure exactly what that would be.

I have always found the blocks intriguing and have been surprised that the 3-, 4-, and 5-year-old children with whom I have worked did not share my enthusiasm. I tried a myriad of ways to stimulate their interest. I bought pattern block cards (see Figure 2) and placed them along with the pattern blocks on a table during activity time. I sat at the table and worked. Children came but they stayed only a few minutes and were off to another activity. As I worked with the materials, I realized the cards were simply a matching activity. After children filled a card or two, there was very little to think about and nothing that challenged them.

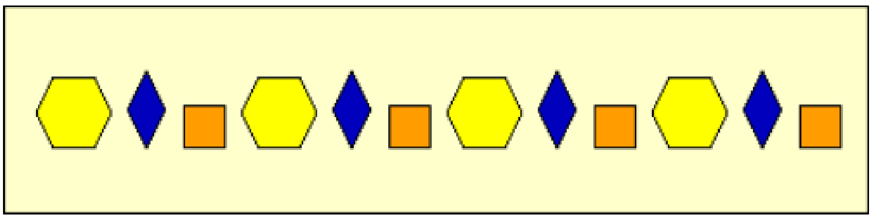
Figure 2 - Examples of pattern block cards



Source: research collection

I spent hours making pattern strips (see Figure 3). I hoped children would match

Figure 3 - Example of pattern block strips

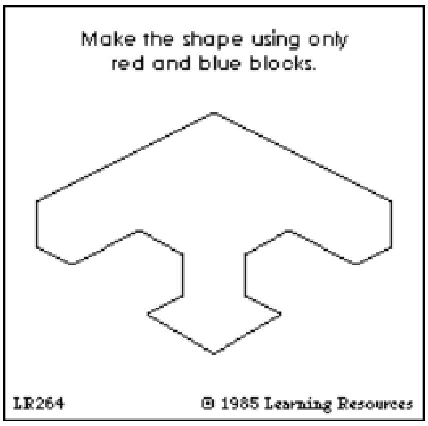


Source: research collection

and extend the patterns I had made and perhaps make their own original patterns. This activity didn't hold their attention any longer than the pattern block cards.

Eventually, I found pattern block cards that had only outlines (see Figure 4) so that children had to figure out what to put inside the drawing. This was better but often

Figure 4 - Example of outlined pattern block card

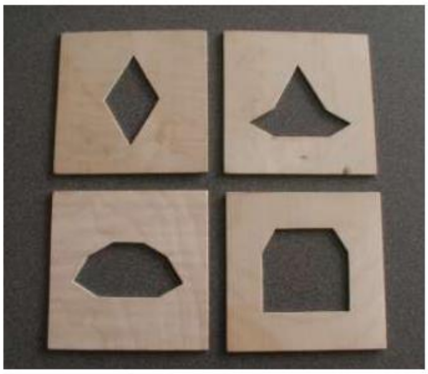


Source: research collection

children appeared not to notice when the pattern blocks slipped outside the lines and their designs did not conform to the outline on the card. They quickly lost interest in these cards as well.

Finally, it occurred to me to make wooden boards similar to puzzles. I created I variety of designs using the pattern blocks and asked a friend to cut holes, in the shapes of the designs, from the center of small rectangles of 1/8th inch plywood. I called them Pattern Block Puzzles. Examples of the puzzles can be seen in Figure 5. When I put the puzzles on the table with the pattern blocks, the children flocked to the table. I could hardly believe it! At last, I had found something that stimulated their interest in the pattern blocks!

Figure 5 - Examples of Pattern Block Puzzles



Source: research collection

As I watched the children, it appeared obvious that they were learning. At first, they demonstrated little knowledge about which blocks to use in each puzzle. After experience, they knew exactly which blocks to use. Because I wanted to know more specifically what they were learning, I designed a mixed method study that included development of a quantitative geometry assessment dubbed the Properties of Shapes Inventory (POSI) which was administered before and after a seven-week intervention.

I selected a constructivist classroom in which children had the opportunity and freedom to choose the pattern block activity during a one-hour activity time. A constructivist classroom was necessary for the study because participation in the pattern block puzzle activity was not required; rather it was available to those who were interested. Further, the activity was designed to encourage children to construct knowledge by testing their own ideas, experiencing the results, reflecting on them, and varying their actions to eventually attain success. The intervention activities were offered to children for 17 days (two or three days a week for seven weeks).

Participants

The entire classroom population (14 children, ranging in age from 4 years 0 months to 4 years 10 months) participated in the study. They attended a federally funded prekindergarten program located in a public elementary school in an urban Midwestern community in the United States. All participants were at or below the federal poverty level requirements and/or had other characteristics that placed them at-risk for school failure. Participants included two African American, one Asian, one Hispanic, one East Indian, and nine Caucasian children.

Setting

The intervention took place within the children's classroom. The pattern block activity was one of the choices available to children during their one-hour activity time. I sat at a table with the pattern blocks and pattern block puzzles. The table accommodated two children and me. Children were free to join and/or leave the activity as their interest dictated. A sign-up list was provided for children to ensure their turn at the activity.

Materials

Materials included pattern blocks, pattern block puzzles, and the POSI pre and posttest.

Pattern Blocks

Four standard sets of 250 pattern blocks composed of six geometric shapes, each shape a different color (see Figure 1), were used in the POSI and the intervention.

Pattern Block Puzzles

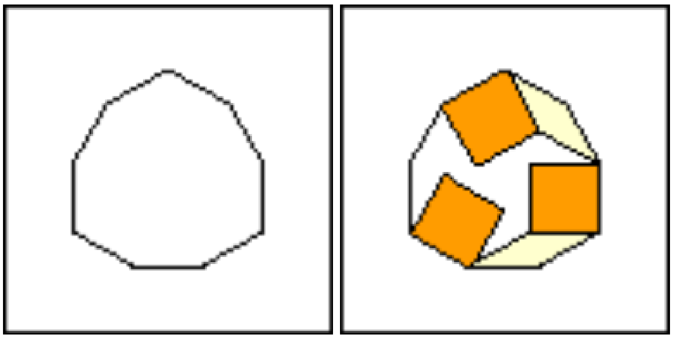
Materials used in the intervention also included 150 pattern block puzzles in a variety of sizes and designs (see examples in Figure 2). They range in size from 2" x 2" to 8" x 10". The cutout space can be filled with pattern blocks in a variety of ways.

Almost all puzzles can be filled in more than one way. The perimeters of some puzzles have only 60-degree and 120-degree angles, thus, requiring only blocks with 60-degree and 120-degree angles. Other puzzles include 90-degree angles and/or

150-degree angles and will accommodate a variety of combinations of all pattern block shapes.

One aspect of the puzzles that challenges children is that pattern block shapes often fit into an angle and into an open area inside the puzzle, but their presence will eventually result in the formation of small odd-shaped spaces into which no pattern block will fit (see Figure 6).

Figure 6 - Example of inserting a block resulting in small, odd-shaped spaces



Source: research collection

Properties of Shapes Inventory

I developed the POSI (SALES; HILDEBTRANDT, 2004), (a protocol of research) to investigate the effects of the pattern-block activities intervention on children's construction of geometric knowledge. It benefitted from work by Clements, Battista and Sarama (2001) who is studying young children's understanding of mathematics, including geometry. He generously shared an unpublished assessment used in testing children (personal communication, January 2001). It was useful as a guide in developing the POSI, the pretest and posttest in this study.

Properties of Shapes Inventory Pretest and Posttest Data Collection and Analysis

A research assistant, with no knowledge of the intervention strategies, administered and videotaped the POSI pretest and posttest assessments. She watched the videotapes of pretest and posttests and scored the tests. I studied data to assess children's progress in constructing geometric knowledge and to search for predictable patterns and strategies they used to solve pattern block problems.

To determine whether the difference between pretest and posttest was statistically significant, t-tests for nonindependent samples were performed on the scores. Effect size was calculated to assess the practical significance of children's gain scores.

Intervention

During group time on the first day, I introduced the materials and showed children the cameras. I presented 41 puzzles of varying difficulty. This allowed children to increase or decrease the challenge depending on their own assessment of their ability to fill a puzzle.

Role of Experimenter During Intervention

As children eagerly came to the pattern block activity, I encouraged them to experiment, observed their actions, assessed their reasoning, and attempted to intervene in appropriate ways to foster their construction of geometric mental relationships. When children's interest waned, I added new and more challenging puzzles to stimulate them to extend their reasoning. If children did not finish puzzles or left the activity in apparent frustration, I added fewer complex puzzles to provide a greater possibility for them to experience success. I added and removed approximately one hundred fifty different pattern block puzzles during the intervention. Figure 7 shows children engaging in the activity.

Figure 7 - Children's shared experience at the pattern block activity

Source: research collection

Intervention Data Collection and Analysis

I videotaped children using two video cameras placed strategically to capture their actions. To ensure the best possible sound quality, I attached a microphone to the table.

To further address the questions concerning children's progress in constructing geometric knowledge and developing predictable patterns and strategies, I conducted a microanalytic study of children's actions. To transform the raw video data into a form that could be analyzed, I transcribed the verbalizations and actions, including facial expressions and other body language, of all 14 children in detail. I invented a variety of symbols and used them in graphic representations to depict specific actions beyond simple insertion or removal of pattern blocks.

The insights into the constructive process from the close observation led to my desire to conduct a theoretical microanalysis that would explain the quantitative findings. I developed criteria for choosing one child as an exemplar and analyzed his actions throughout the intervention to understand his learning process.

Making Inferences about Cognition from Behavior After choosing Noah, I studied the videotapes and the detailed transcripts of his verbalizations (e.g. words, groans, and tone) and gestures (e.g. hand movements, facial expressions, and other body language such as walking away, returning, reaching out over the pile of pattern blocks, choosing a pattern block shape, inserting, rejecting, hesitating, looking from the puzzle to the pile of pattern blocks repeatedly, bouncing, leaning back and forth against the table, and patting his designs). This analysis provided information about his cognitive progress, as well as his affective state. I searched for evidence or lack of evidence of contradictions to expectations resulting in disequilibrium, decentering to consider other perspectives, construction of new mental relationships, and reequilibration. I used Piaget's theory of equilibration as a framework to understand what Noah's actions indicated about his reasoning. My detailed analysis using Piaget's theory (PIAGET, 1936/1952, 1937/1954, 1975/1985; PIAGET; GARCIA, 1991; PIAGET; INHELDER, 1948/1956) as aframework, provided insights about his behavior from which I made inferences about his cognitive development.

Included in the results of the study are transcripts with drawings of Noah's movements and descriptions of his error-informed experimentation, construction of mental relationships with misconceptions, expectations formed based on his mental relationships, contradictions to expectations and resultant disequilibrium, reequilibration through construction of new mental relationships, how the meaning of the pattern blocks and puzzles changed for him, and the role of affect during the process.

Summary of Pretest and Posttest Results

The identical POSI pretests and posttests provided information to answer both questions of this study: (a) what, if any, progress in constructing geometric knowledge do four-year-old children show after a short-term intervention, and (b) are there predictable patterns or transitions that emerge in young children's ability to solve pattern block problems? If so, what are they?

The difference between the means of the POSI pretest and posttest scores was found to be highly significant, $t(13) = 6.68$, $p < .0001$. These results, along with the effect size of 1.79 are evidence that children's increase in knowledge about the properties of shapes and spaces after the intervention according to the POSI is both real and large. Findings from an analysis of puzzle configurations indicate that the more acute the angle, the more recognizable it was to the children in this study. In the pretest, the 14 children made 147 correct predictions about which shapes fit into corresponding spaces. In the posttest, this figure increased to 328 correct predictions.

Children learned to match shapes with corresponding spaces, distinguishing among, and coordinating the sizes of varied angles and spaces, and substitute smaller blocks for larger blocks. Preventing children's interaction with pattern blocks and similar materials before the study and the short duration of the intervention helped control for internal validity problems such as maturation and a history of experience. Therefore, it seems reasonable to conclude that children's construction of geometric knowledge was due to the pattern block intervention.

The significant POSI test results provided a general overview of children's progress in geometric reasoning. From a Piagetian perspective, the pretest and posttest results indicated that children constructed a complex network of mental relationships enabling them to match sizes of angles in spaces to angles of blocks, match the area required by a specific pattern block to the area available inside a puzzle, understand the substitution relationships among various pattern blocks, and act mentally without acting physically when they made predictions about which pattern blocks to use to fill the puzzles. More children were able to match angles of spaces to angles of blocks than could match area required by a block and the available space inside the puzzle. This finding suggests that children constructed angle relationships before they constructed area relationships. The detailed analysis of Noah's actions using Piaget's theory (PIAGET, 1936/1952, 1937/1954, 1975/1985; PIAGET; INHELDER, 1948/1956) as a framework supported these findings and provided information about his transitions as he constructed geometric knowledge.

The findings lend credence to Montessori's (MONTESSORI, 1912/1964, 1914/1965), Froebel's (KRAUS-BOELTE; KRAUS, 1882), and Piaget's (PAIGET, 1936/1952, 1937/1954, 1975/1985; PIAGET; INHELDER, 1948/1956) ideas that young children learn about their world by acting on objects that are interesting to them and to Froebel's claim that young children learn about geometric shapes by changing the object's form, that is, composing, decomposing, and rearranging it. However, in contrast to Montessori's and Froebel's prescripted steps to teach children about geometric shapes by avoiding errors, the actions of the children in the present study, as Piaget (PIAGET; GARCIA, 1991) found in other contexts, were fraught with errors. These errors informed me of the misconceptions that children did indeed confront, examine, and eventually discard during their work constructing new and more adequate mental relationships.

Summary and Discussion of the Qualitative Microanalysis

More than any other child, Noah's actions appeared to be indicative of his construction of geometric knowledge. He received the lowest pretest score indicating that he knew little about pattern block shapes before the intervention. However, throughout the intervention, he consistently challenged himself by choosing puzzles that were difficult for him. In contrast to his lack of interest in the pretest, Noah's high interest, persistence, and gradual mental progression as he dealt with problems, he encountered in the pattern block activity provided an abundance of observable clues to his reasoning during his construction of geometric knowledge. His level of reasoning and knowledge changed dramatically over the course of the seven-week intervention.

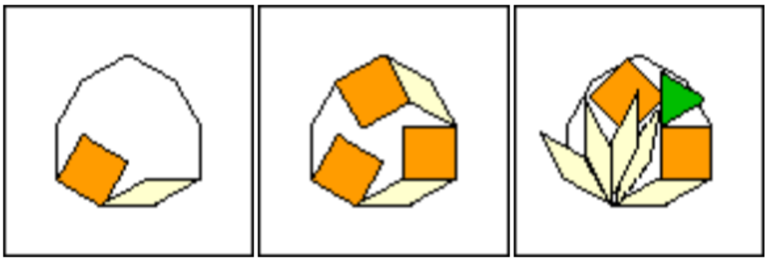
According to Piaget (1932/1963), the development of intelligence involves the mental action of modifying existing mental relationships and reorganizing them into new mental relationships. I studied Noah's physical actions searching for evidence of his mental construction of geometric knowledge. Piaget's research and theory suggested a method of understanding Noah's actions in terms of inventing, organizing, and elaborating his mental relationships.

Over the course of the intervention, Noah chose to work on 258 puzzles, of which I selected those that demonstrated his beginning point and evidence of intellectual development. Noah constructed an increasingly complex network of mental relationships as he confronted his errors, decentered to consider more than one aspect simultaneously, and eventually considered alternative solutions to problems presented by the puzzles.

The analysis supported Piaget's equilibration theory. Noah constructed corresponding mental relationships between the angles in the spaces inside the puzzles and the angles of the pattern block shapes, and mental relationships between the amount of space required for each pattern block shape and the amount of space available inside each puzzle. A great change occurred in Noah's reasoning when he finally accepted the limitations imposed by the puzzles, and his network of coordinated relationships became increasingly more adequate and powerful. These newly-formed and more numerous and complex mental relationships constituted the progressive development of Noah's intelligence as well as his knowledge.

There seemed to be an order in which Noah constructed new relationships. First, he constructed relationships among the 30 degree and 60-degree angles of the empty spaces inside the puzzles and the 30 degree and 60-degree angles of the block shapes. Initially, Noah did not coordinate the size of the available area with the size of the pattern block to be inserted as seen in Figure 8.

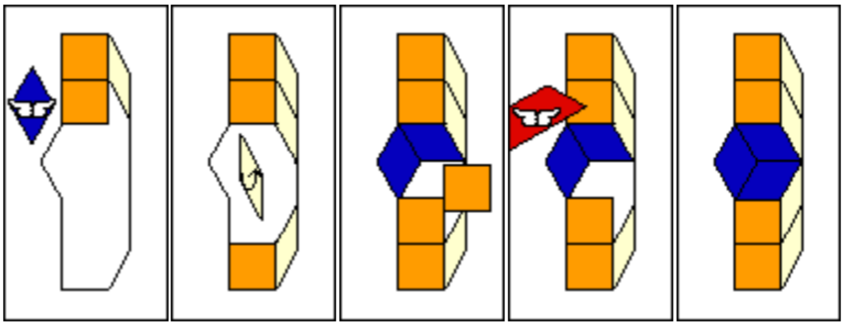
Figure 8 - Examples of moves in nonagon puzzle (Day 1, Puzzle 13, 14 Moves)



Source: research collection

By Day 4, he began picking up blocks, moving them toward empty spaces, and returning them to the table before attempting to insert them. This action indicated he was considering the size and shape of the empty space relative to the size and shape of the block. Eventually, he began to construct mental relationships among larger angles. On Day 7, he demonstrated a stable relationship between the 90-degree angles and the square (see Figure 9).

Figure 9 - Examples of moves in elongated puzzle (Day 7, Puzzle 65, 15 Moves)



Source: research collection

The following is a conceptualization of characteristics describing aspects of Noah's constructive process as evidenced by his actions.

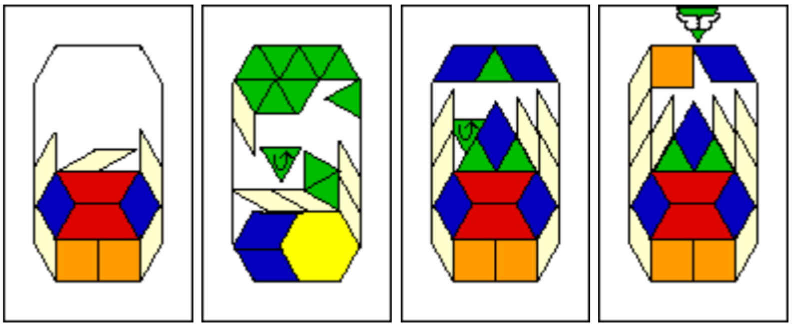
Characteristics of Noah's Constructive Process

- Noah comes to the activity with some prior mental relationships about shapes.
- Noah begins constructing new relationships about the meaning of both the pattern blocks and the pattern block puzzles during his initial contacts with them. (This constructive process continues throughout the intervention.)
- Noah's actions on the blocks and puzzles are based on his expectations about what and how they will fit into the puzzles.
- Noah experiences contradictions to his expectations when blocks do not fit as he expects, which results in psychological disequilibrium—cognitive and affective.

- When Noah's disequilibrium is so great that he is overwhelmed cognitively and emotionally, the constructive process is thwarted, and he abandons the problem.
- When Noah's expectations are met, he experiences satisfaction and confidence in his ability to solve pattern block puzzle problems.
- Following success, new confidence appears to lead to tolerance for disequilibrium and reduction of frustration.
- Noah engages in error-informed experimentation.
- Noah's constructed network of relationships enable him to decenter and view the pattern blocks and puzzles in terms of multiple aspects (angles and area).

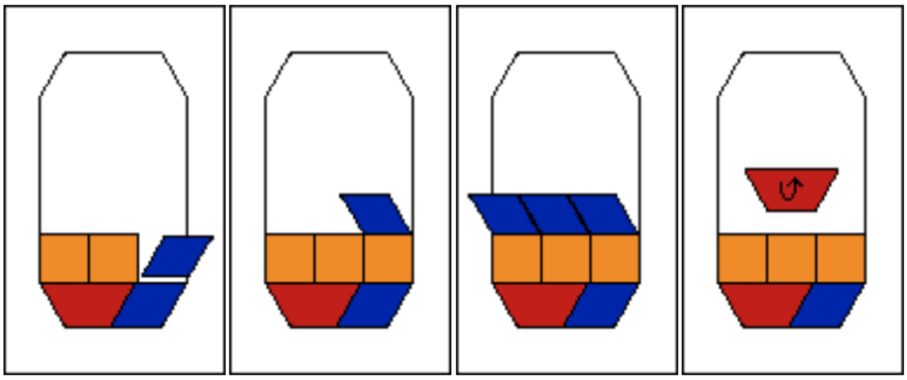
During Noah's constructive process, in the beginning he was hindered by his inability to decenter. Like most children his age, Noah tended to focus only on one perspective at a time. Sometimes Noah's centrations prevented him from accessing what he already knew. For example, his centration on white rhombuses and triangles on Day 5 (see Figure 10) and on blue rhombuses and trapezoids on Day 7 (see Figure 11).

Figure 10 - Examples of moves in elongated octagonal puzzle where Noah has ignored the 90-degree angle (Day 5, Puzzle 44, 176 Moves)



Source: research collection

Figure 11 - Examples of moves in elongated octagonal puzzle where Noah has ignored the 90-degree angle (Day 7, Puzzle 61, 43 Moves)



Source: research collection

Prevented him from drawing upon the 90-degree-angle relationship he was in the process of constructing. When that mental relationship, that is, the mental relationship between the 90-degree angle of a space and the 90-degree angles of the squares became stable (in other words, when that knowledge was solid), he could fill almost any puzzle despite its difficulty.

Noah frequently asked for help from me while he was attempting to insert blocks that could not fit because of the configuration of the design. When I made suggestions to help him decenter and look at the design from different points of view, he repeatedly rejected my help. Asking for help and then rejecting it seemed illogical and caused me to experience disequilibrium. However, further reflection of Noah's behavior revealed a possible explanation. It appeared that Noah had an attachment to his ideas that was both cognitive and emotional. Not only did Noah fervently want his ideas to work, he believed they would! From his first experiences, he appeared to conclude that if a block fits into an angle and inside a puzzle, it belonged, and he forsook any evidence to the contrary. When I was intervening, I understood Noah to be asking for help to figure out how to fill a puzzle without any gaps. I now believe that he was actually asking for help to make his ideas work without regard to the limitations imposed by the design of the puzzle. He was not asking me to help him see that his ideas would not work. Because my suggestions were aimed at changing his ideas, he, not so illogically, rejected them. In fact, he rejected any help that would have caused him to change his ideas.

This conclusion has led to me to believe that before Noah could construct new cognitive knowledge (a stable mental relationship between the 90-degree-angled space and the 90 degrees of the angles of a square), he had to decide for himself to abandon his emotional attachment to the ideas he so badly wanted to work. He was only ready to do so after he tried again, and again, and again, and again, proving to himself that his ideas would not work. Only when he was totally discouraged was he willing to look at another possibility, in other words, to take another perspective. Only when he was open to another perspective was he not only willing but able to allow himself to consider and construct a mental relationship between the 90-degree-angled space and the 90 degrees of the angles of a square. I believe that Noah's struggle to make his ideas work, eventually leading to his conclusion that those ideas simply would not work and his acceptance of the necessity of considering another point of view, played a major role in his journey to construct new knowledge.

This analysis of Noah's request for help has changed my understanding of appropriate teacher intervention. As I worked with him, I intended to help him take other perspectives by suggesting different ways he might fill the puzzle. It now appears that the focus of my intervention should have been to help him understand that his ideas would not work rather than to make suggestions about what he should change to enable him to fill the puzzle. For example, sometimes I suggested he remove blocks. Those suggestions were met with a resounding "Noooooooooo!" However, when I made comments that drew his attention to the results of his actions (e.g., "It leaves that funny space there doesn't it."), Noah focused more on what was working and what he needed to change than on rejecting my suggestions.

Noah's errors were a vital part of his construction of knowledge and intelligence. Often when he inserted blocks, the odd shaped spaces left inside the puzzle were too small to accommodate any other pattern block. In the beginning, Noah seemed unaware of the too small spaces he created. He continued to insert blocks with angles matching angles inside the puzzle but with spaces too small (see

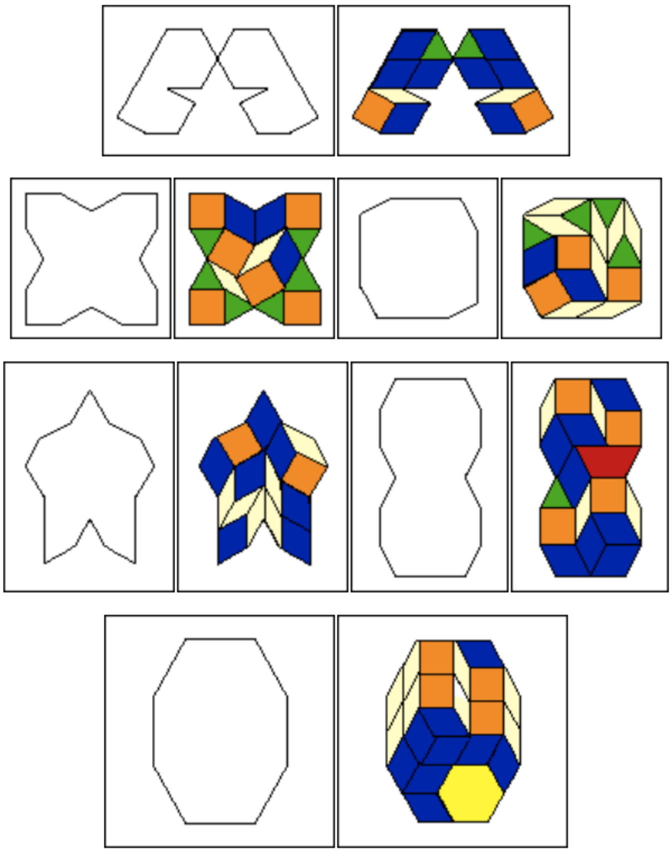
Figure 8). However, the perimeter of the designs and shapes of the spaces inside the puzzles were a rigid and consistent source of feedback. When he attempted to insert blocks into the too small spaces, the blocks protruded over the edge of the puzzle. After he repeatedly experienced the results of such errors, his shoulders slumped, and he appeared to be discouraged and downhearted. This was a clear sign of great disequilibrium. It was only then that Noah was finally willing to let go of his attachment and open himself to another perspective.

According to Piaget (1981), affectivity is the energy source upon which the functioning of intelligence depends. In other words, affectivity stimulates or hinders the construction of mental relationships. Noah's attachment to particular blocks was hindering his ability to construct new relationships. Only after he repeatedly experienced contradictions to his expectations (disequilibrium), did he finally shed his attachment, consider the results of his physical actions, and reason about other solutions. He placed new ideas into relationship with old ideas, compared them, and modified existing mental relationships about space. As the process continued, he constructed more complex and adequate mental relationships.

Noah took great pride in his accomplishments. He proclaimed himself "clever" and "good." No one had to build up Noah's self-esteem; he built it for himself. Noah constructed not only knowledge and intelligence but also a confident belief in his ability to reason.

During the last ten days of the intervention, Noah needed very little assistance. His predictions about which pattern blocks would fit into which spaces became increasingly more accurate. With very few errors, he could fill almost any puzzle. He seldom selected blocks he could not insert accurately, and he continued to challenge himself by selecting increasingly difficult puzzle with large open areas requiring all five angles (see Figure 12)

Figure 12 - A variety of puzzles Noah filled without difficulty (Days 8-17)



Source: research collection

The significant results of the POSI and the detailed analysis of Noah's experience are evidence that four-year-old children constructed geometric knowledge and that predictable patterns and transitions emerged after the short-term intervention.

Educational Implications

The mathematics standards written by National Council of Teacher of Mathematics (2000, p.57) recommend that young children "analyze characteristics and properties of two- and three-dimensional geometric shapes, apply transformations and use symmetry to analyze mathematical situations, and use visualization and spatial reasoning." The activity in the study is one way to address these recommendations. The results suggest that when engaged children rely on their own reasoning to figure out solutions to geometric problems that interest them, they will construct geometric knowledge. When they have the opportunity to try their ideas, make errors, experience the results of those errors, they modify and reorganize existing mental relationships. Through this process, children's internal beliefs evolve. They construct increasingly complex mental structures providing power to solve new problems.

Teacher interventions should focus on helping children analyze their own actions rather than making suggestions about what they might try. This study suggests that children change what they believe to be true by confronting their misconceptions, not by avoiding them. When educators provide children with solutions, they deprive children of the opportunity to struggle with their misconceptions, change their beliefs, and devise their own solutions. By figuring out solutions or correct answers for themselves, children not only know correct answers, they understand them. Perhaps we adults don't trust children to do this. This study suggests children can and do.

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